Original article

Predicting the limits of human athletic performance

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Abstract

We investigated the limits to the athletic performance of humans: how fast can we run or swim, how high or long can we jump, how far can we throw? We proposed a mathematical model both to model the improvements in the performance and to predict the limits. Using data obtained from the Olympic games over the past century, we found that our model worked across all the athletic disciplines considered, including running, swimming, jumping, and throwing events. The model required a small (between 2 and 6) set of parameters, and using goodness-of-fit metrics, we showed that our results had a high degree of accuracy and were measurably superior to prior results presented in the literature that used mathematical modeling. Our results also agreed well with those obtained by researchers using biomechanical approaches, such that further exploration using a joint computational and biological approach may prove worthy of perusal.

Keywords

Athletic performance, Mathematical model, Limits of athletic performance, Olympic games, Predicting athletic performance, Athletic records, Athletic performance prediction, Running, Jumping, Swimming

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1. Introduction

There have been several previous efforts to model the improvements in athletic performance and to predict the limits of that performance. In (1), a linear regression model was used to extrapolate the speed which runners could achieve. The analysis resulted in the absurd result that eventually race times became negative. This is a classic case of using a poor fitting function. Similarly, (2) used a linear model and extrapolated illogically, which also led to impossible conclusions. As a result, (2) not only faced the same problem as (1), but by comparing the trends of athletic performance among genders, concluded that women would eclipse men's records in a few years. This ostensibly fallacious conclusion arose because women's performance has improved at a faster rate than men's in recent years.

The limitations of the approaches by (1) and (6) are noted in (3). They proposed an exponential function of the form $\frac{e^{at}}{1+e^{at}}$ which resulted in an S-shaped curve and fit the data to this curve. The limitations of this approach are (1) the goal was to fit every data point, regardless of vintage, which could result in an overfitted model, that in turn, may not be a good predictor, and (2) they used only world record times. Data from 1920, when amateur athletes from a select subset of countries participated, were given the same weight as world records in the late 20th century. There are several papers which investigate this from a bio-mechanical perspective. In particular, (1) discussed the limitations of athletic achievement and the specific causes – technology, morphology, phenotype, socio-cultural impact, environment, economics, and finally randomness. The paper also made the case for considering periodic tournaments (such as the Olympics) rather than world records. For example, in the case of Bob Beamon, it led to a 22-year drought of world records in long jump. Weyand et al. and Graubner et al. performed biological and bio-mechanical analysis on this problem (5, 6).

This paper used the Olympics as the periodic tournament to model and predict athletic performance. The rest of this paper is organized as follows: we first outline our approach to the problem, including the justification for the data used, and our mathematical models. We then outline our assumptions, including limiting the degrees of freedom of the functions used, the cases in which asymptotic limitations are justified, and the conclusions that can be drawn in these cases. We then present results for men and women's running events, swimming, throwing (discus, shot put, and javelin), and jumping (long jump). We also present examples (women long distance running, men's high jump) where either due to lack of data or other reasons, our model is not a good predictor. We then discuss the results and show the connection between our results and those obtained in the literature using bio-mechanical approaches. Finally, we conclude with some thoughts on areas for future work.

2 Methodology

We have focused our data on athletic events held at the Olympics. We describe below the events that we have chosen, and the rationale for the tournaments from which we gather our data. Following this, we present the mathematical approach that we take -- first outlining the parametric model that we use, and our approach to weighting the data to improve our prediction. Finally, we outline the way in which we present our results, including the standard error and the goodness of fit metrics that we use.

2.1 Events

For this initial research, we limited the events that we analyzed to those which involved speed (running, swimming) and strength (in particular, throwing events including javelin, discuss, and shot put). These events have been present in tournaments for a long time, use minimal equipment, and are widely contested. Specifically, we used - Running: 100m, 200m, 400m, 800m, 1500m, 5k, 10k, marathon. Throwing: discus, shot put, javelin. Swimming: 100m, 400m. In the swimming category, the events have not been held consistently. Consequently, we limited the events we studied to 100m and 400m for men.

We modeled results from both men and women. However, the data from women's competitions was somewhat uneven -- for example, in several running events (including 400m, 1500m, and for all events greater than 1500m) at the Olympics, data only exists from the 1950's, and in some cases only since the 1980's. Men's events typically had consistent data for over 100 years.

2.2 Tournaments

We focussed on winning performances at the Olympics. This provided a rich trove of high quality, periodic data. The Olympics have been held regularly (with the following exceptions: 1940 and 1944 due to WW2 and 2020 due to the pandemic). No other tournament or competition has either the longevity or the consistency. Additionally, there was the advantage of periodicity – using world records would not only be sporadic, but a single freak performance (such as Bob Beamon's long jump record in the 1968 Olympics) could create a barrier for additional records for decades; in this case, taking over two decades. Finally, the world championships were another good source of periodic data, but they have only been in existence since 1976. Overall, the Olympics were deemed to be the best source of data.

2.3 Mathematical Model

2.3.1 The general model

We used a rational polynomial to model the progression of athletic performances in the past century. A rational polynomial, $f(x)$, can be represented as a ratio of two polynomial functions, expressed as equation 1 below.

$$f(x)=\frac{g(x)}{h(x)}=\frac{g_0+g_1x+g_2x^2+\cdots}{h_0+h_1x+h_2x^2+\cdots},$$

where the numerator, $g(x)$,and the denominator, $h(x)$, each has an infinite number of terms. This is a very general expression and can be used to model virtually any function. We now define the finite-term version of the rational polynomial given by equation 2 below.

$$f_N(x)=\frac{g_N(x)}{h_N(x)}=\frac{g_0+g_1x+g_2x^2+\cdots+g_Nx^n}{h_0+h_1x+h_2x^2+\cdots+h_Nx^n}.$$

We shall use this (finite-term) rational polynomial to model athletic performance. The goal is to use the lowest N.This is desirable for two reasons. First, we wish to have the simplest explanation to any question, which is the least complex function to explain a phenomenon. Second, it prevents overfitting. We do not wish to include more parameters than can be justified by the data.

2.3.2 Finding the Asymptote

The best fit function is parametrized by $(g_0,\ldots,g_N,h_0,\ldots,h_N)$ in equation 2 above. If we can rewrite the function as $f_N(x)=A+f'_N(x)$, and that $\lim_{x\to\infty} f'_N(x)=0$, this results in equation 3, $\lim_{x\to\infty} f_N(x)=A$. In those cases where equation 3 holds we can conclude that $f_N(x)$ has a limit. In the cases of speed-related events (track, swimming, etc.) this is the asymptotic speed limit, S_ULT. In distance-related events (discus, shot put, javelin, high jump, and long jump) this is the asymptotic distance limit, D_ULT.

2.3.3 Weighting the Data

In the past century, sporting events have become more widespread and democratized, worldwide participation has increased greatly, and athletes' training regimen and focus has steadily improved. Additionally, as discussed earlier, many events were not even competed for in the first few decades of the 20th century. With this background, we considered weighting the data so that recent events were given more importance than past events. This was accomplished by making the data "noisy" and assigning a value to the noise for each data point. The noise (i.e., the standard deviation, σ, associated with the data point) that we assigned was a function of how old the data was, and the noise was modeled as being proportional to $\frac{1}{t+c}$, where t is the age (in years) of the data and c is a small, positive constant. This was somewhat arbitrary and only had a modest impact on the final results. We used it to fine-tune the non-linear optimization function to ensure that more recent data were given greater importance. The model was used to analyze historical data and predict future performances, and the associated asymptotic limits.

Over the past century there have been many changes in each of the sports that we studied: technological advancements (including in shoes, clothing, and equipment), rules, nutrition, and training of the athletes, etc. The only way in which we accommodated these was by incorporating the uncertainty of the data as a function of its age (i.e., by increasing the noise variance, as described above, for older data). Other than this, the data were not adjusted or normalized in any other way.

2.3.4 Validating the Model

To validate the accuracy of the model, we used three approaches. First, we used the goodness of fit measure, the Coefficient of Determination, R^2 to calculate how well the dependent variable (e.g., either speed or distance) was predicted by the model. However, it should be noted that R^2 is more appropriate for linear models. We provided it anyway, as one manner of validation, but augmented this with the second approach, which was to use the standard error, σ_{SE}. The standard error is the square root of the estimated covariance of the calculated coefficients. A small σ_{SE}. compared to the estimate, was indicative of a good model.

Furthermore, if we assume that the data are distributed in a roughly normal (Gaussian) distribution, we can say that a band provided by $S\_ULT \pm 3\sigma_{SE}$ (or $D\_ULT \pm 3\sigma_{SE}$) will provide a high-confidence (99%) bound for our estimates.

Finally, we compared our results with those obtained from biomechanical approaches such as (1, 6), and (5). These efforts are still in their early stages but provided a useful template for our mathematical approach.

3 Results

We first considered one event to delve more deeply into the model. Accordingly, Figure 1 shows the results for the men's 100m dash at the Olympics.

As discussed in the modeling section, if the function which describes the model has a limit as time goes to infinity, then our model could establish the asymptotic speed. That was, in fact, the case here, and we found that the asymptotic speed was 37.5 km. We also calculated σ_{SE}, the Standard Error. We found that σ_{SE}=0.25 km/hr, which is quite low and indicated that the function we calculated was a good fit to the data. The coefficient of determination, $R^2 \approx 0.84$,also indicated a good fit.

Finally, using the asymptotic value and the standard error, and assuming that the distribution is normal (which is a working hypothesis but one that we do not defend) we designated the region $S_{ULT} \pm 3\sigma_{SE}$ the Ultimate Performance Band with confidence greater than 99%. Our assertion is that the ultimate speed limit for humans running the 100m dash lies between 36.8 km/h and 38.2 km/h.

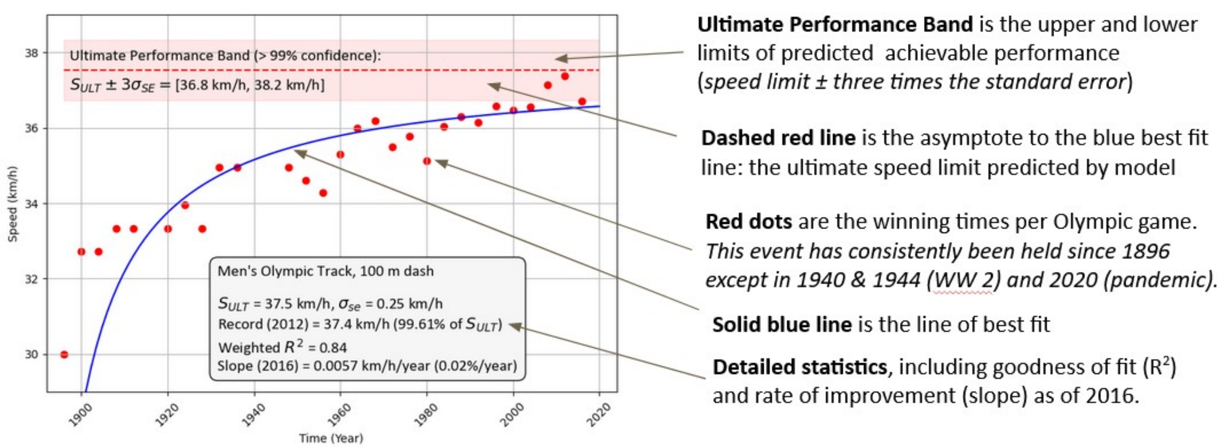
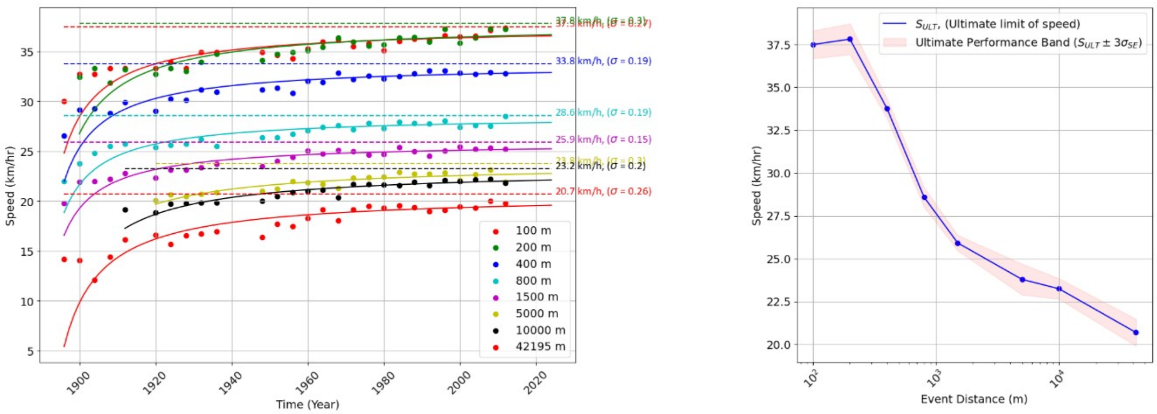


Figure 1. Men's 100m dash at the Olympics

3.1 Running

Next, we analyzed all the running events, from 100m to the marathon. The combined results are shown in Figure 2, and to prevent overcrowding we show the asymptotic results in figure 2(a) and the performance bands in figure 2(b). It can be seen that the standard errors are small, resulting in tight performance bands, indicating high confidence in the estimates.

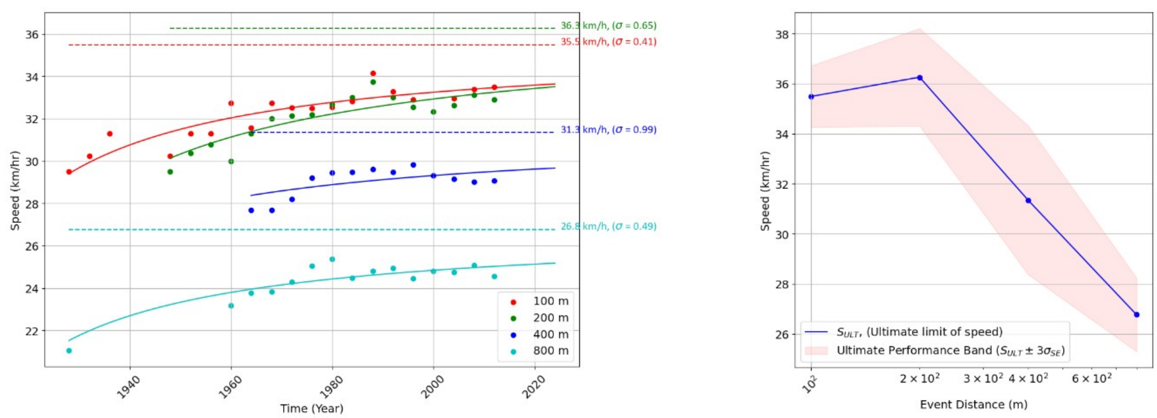


(a) Asymptotic speed limits for men's running

(b) Performance bands across distance events

Figure 2. Men's Running events, winning speeds at the Olympics from 100m to the marathon

Next, we show similar results for women's running. The data was restricted to only four events: 100m, 200m, 400m, and 800m. Figure 3(a) shows the detailed times, model, and asymptote, and figure 3(b) shows the performance limits across the distance events for women. Here, again, the standard error was reasonably good, although not as good as for the men's events.



(a) Asymptotic speed limits for women's running

(b) Performance bands across distance events

Figure 3. Women's Running events, winning speeds at the Olympics:100m - 800m

The Women's longer distance running events were not held consistently at the Olympics, and many of the longer distance running events were not held at all until the 1970's.

Furthermore, there was an additional problem. The data was not only sparse, but the performance times got worse with time. One possible reason is the widespread use of performance-enhancing drugs among women athletes (e.g., East Germany) which makes many of the records set in the 1960s to the 1980s suspect. Figure 4 shows the results for women's 1500m.

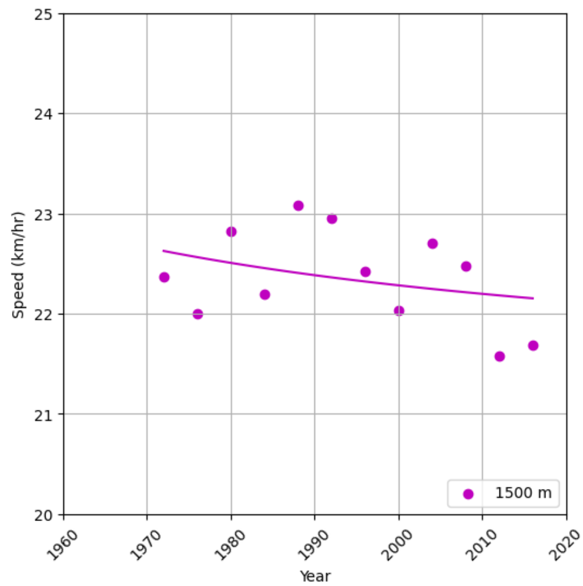
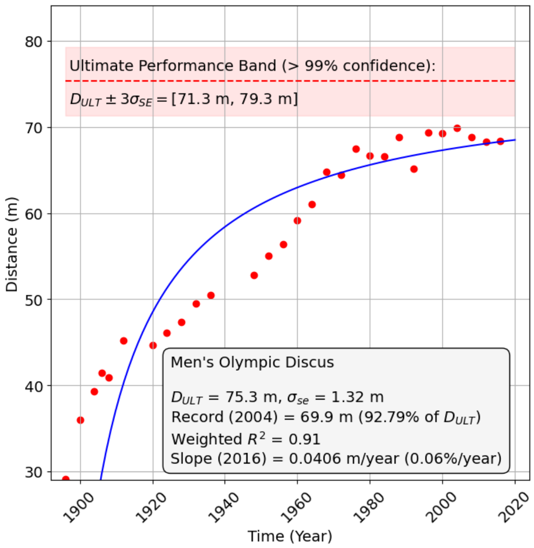


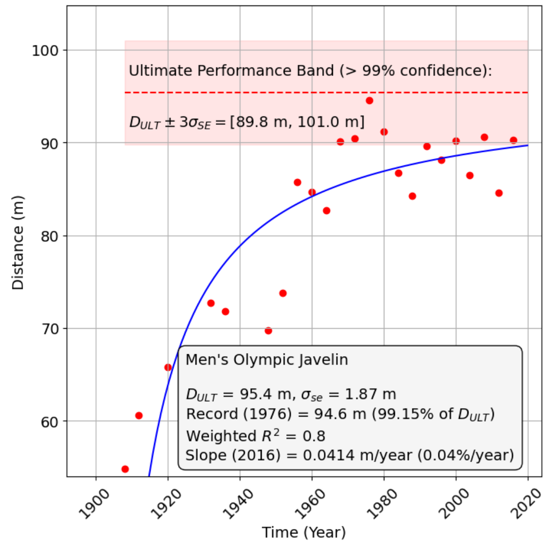
Figure 4. Women's 1500m event at the Olympics

3.2 Throwing

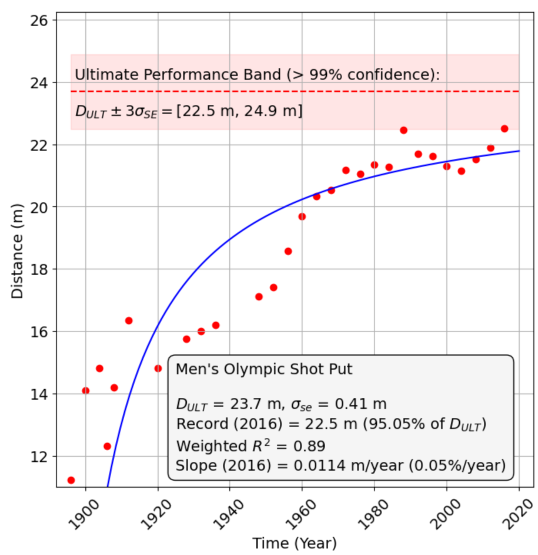
We next consider some throwing events: discus, javelin and shot put. All three events presented with good data from the very first modern Olympic games, 1896 to the present, and could be well modeled. Results are presented in Figure 5. It can be seen that the model explained the data well; there was a good fit without overfitting. For all three events, the goodness-of-fit metric was high (>0.8), and the standard error was bounded and provided intervals of high confidence.



(a) Discus



(b) Javelin



(c) Shot Put

Figure 5. Men's Throwing events, winning distances at the Olympics

3.3 Jumping

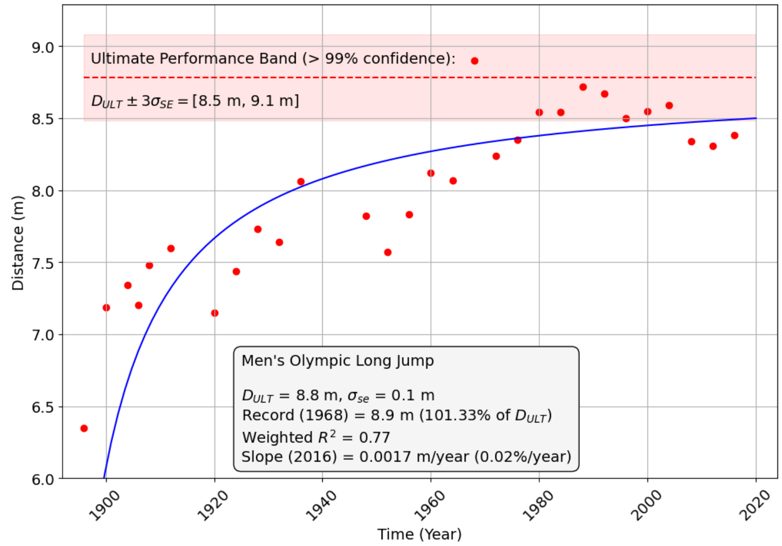


Figure 6. Men's Olympic Long Jump event

Two jumping events were considered, long jump and high jump. The long jump could be well modeled, as seen in Figure 6. This is despite Bob Beamon's remarkable jump at the 1968 Mexico City Olympics, which may have been a freak record that stood for more than two decades. Nevertheless, the power of the model was such that we could see the ultimate performance, and that Bob Beamon's record was still within the performance band predicted.

The high jump event posed an interesting question. In Figure 7, it can be seen that over the past 40 years at the Olympics, the winning times have been almost constant. Possible reasons for this include a lower dependence on technology and equipment, and that techniques have not changed significantly since the introduction of the Fosbury Flop in the 1960s.

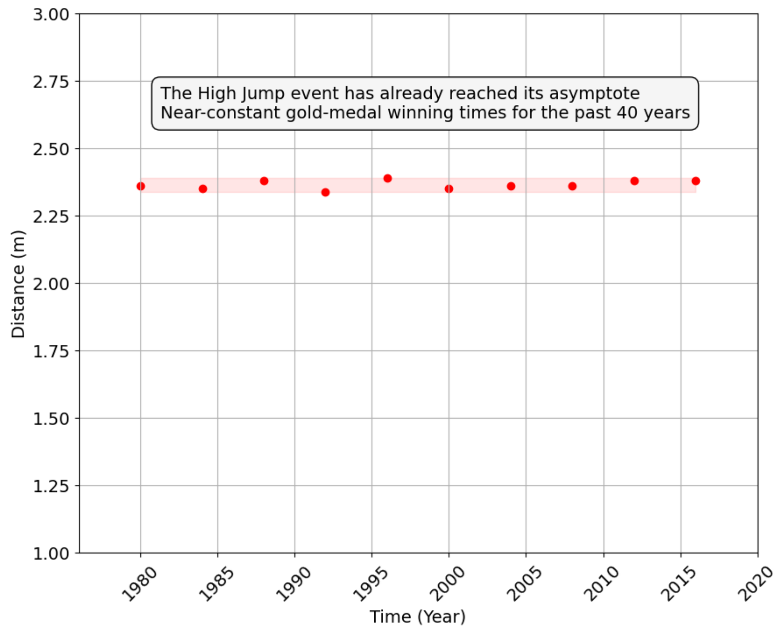
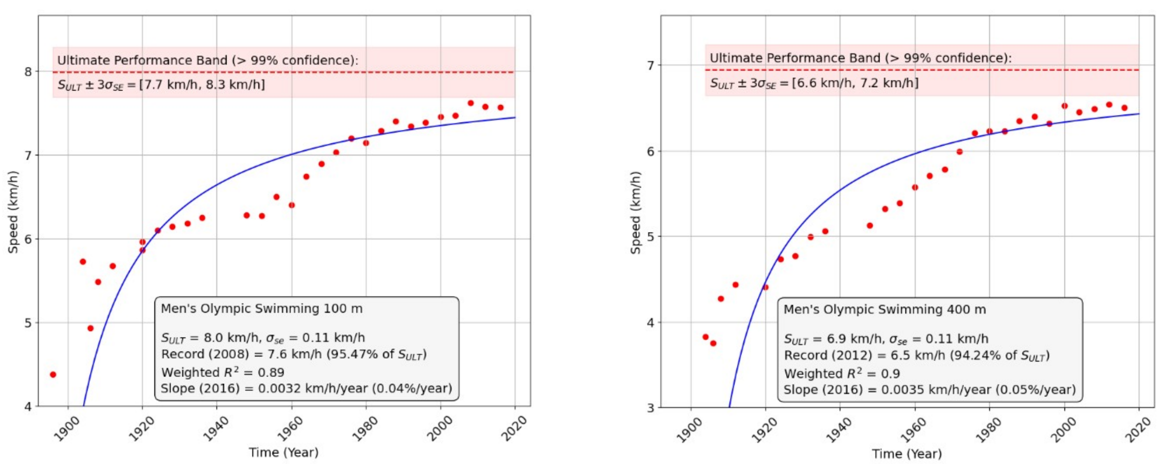


Figure 7. Men's Olympic High Jump event

3.4 Swimming

The swimming events investigated also have good, consistent data which lended themselves to being well modeled. Figure 8 presents the results for men’s 100m and 400m freestyle at the Olympics.



(a) Men's 100m freestyle

(b) Men's 400m freestyle

Figure 8. Men's Swimming events, winning times at the Olympics

4 Conclusions

For a range of events, the accuracy of the model could be validated mathematically and compared to previous literature. The model fitted the data well across a variety of events in the speed and distance categories. Only in events in which there was poor data (e.g., some of the women's longer track events where there was declining performance) or where an asymptote had already been reached (high jump) did the model not produce good results. The coefficient of determination, R^2, ranged from 0.8-0.9 (a perfect fit is 1.0) in almost events. This indicated a good model with high prediction accuracy. Additionally, the standard error, σ_{SE}, was small relative to the estimate, being indicative of a good model. When compared to prior literature on this subject, the model proposed in this paper was found to be superior to those proposed in (2-4). It provided higher accuracy, and because it had few parameters there was lesser chance of overfitting. There were no strange results such as negative times.

Comparing the results of the model with literature which approaches this problem from a biomechanical perspective is a useful exercise. In this regard, (2) provides a good methodology to study running from the physiological approach. The paper proposes that the limit of a runner is based on their strides. Higher speeds were achieved when athletes put more force on the ground in a shorter amount of time. The less amount of time an athlete spends in the air, the faster they have to swing their limbs, thus increasing their speed. Top athletes all move their limbs at a similar rate, but the difference in their speed is caused by the amount of force they can apply on the ground. Based on this, the paper provided a range of predictions about what the top speeds would be, ranging from a low of about 35 km/h to a high of 50 km/h. However, the authors admitted that the upper limits of this range were based on athletes having much longer limbs, and no inhibitions of movement, etc. Consequently, if one restricts the estimate to those at the lower range, this model compared well with our top speed estimate for men’s 100m dash of 36.8 km/h to 38.2 km/h. This approach to refine the estimates of best possible speeds based on biomechanical perspective may be an interesting area for future work.

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