



Dimensionality reduction of the freedoms constituting The Index of Economic Freedom.

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Abstract

The Index of Economic Freedom, published by the Heritage Foundation, ranks countries according to their performance in terms of twelve economic freedoms. These twelve economic freedoms are in turn divided into four categories; rule of law, government size, regulatory efficiency and open markets. The Index is computed by taking an average of all the twelve economic freedoms, which assumes that each individual basic freedom contributes equally to the Economic Freedom Index. Principal Component Analysis (PCA) was used and employed on a matrix composed of the twelve economic freedoms in order to perform dimensionality reduction. The result indicated that the first two principal components could capture 62.9%, or $\sim 2/3^{\text{rd}}$ of the variance in the data for 184 countries. Examination of the eigenvector values of the PCA indicated that the tax burden and business freedom were approximately orthogonal attributes and plotting one against the other captured a variance comparable to that obtained from the first two principal components. This result was verified by using a process of orthogonality detection using the correlation data between the principal components and the twelve economic freedoms obtained from an online PCA calculator. It may thus be possible to obtain an accurate relative ranking of the economic freedom of individual countries by using only two of the twelve economic freedoms. This greatly simplifies, reduces the expenditure and effort, and increases the speed, of data collection, analysis and presentation. This may, in turn allow for a significant adoption expansion of the Index as *the* barometer of Economic Freedom. It may also allow other suitable multifactorial indices to be depicted relatively accurately using reduced dimensions.

Keywords

Principal component analysis, Index of Economic Freedom, Heritage Foundation, Economic freedom, Tax burden, Business freedom, Orthogonality, Dimensionality reduction, Variance

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Introduction

The Index of Economic Freedom measures twelve economic freedoms in each country and ranks 184 countries in terms of how economically free they are. The Index hence provides a first approximation of the relative economic freedom in a country. A higher level of economic freedom in a given country has been linked to stronger economies and higher GDP, which in turn create a higher standard of living and education for its citizens. In addition, a result of a freer economy is free trade, which allows a country's economy to prosper. In general, a freer economy is beneficial for individuals and to society. Therefore, examining how the different freedoms contribute to the holistic term of "economic freedom" can provide a better understanding of how they improve the well-being of a country's citizens and its overall impact (1).

The Index of Economic Freedom, assigns equal weight to the twelve components of economic freedom so that the overall score is not biased toward any one component or policy direction. The Index compilers recognize; and state in the methodology section at their website (2) that, "It is obvious that the 12 economic freedoms interact, but the exact mechanisms of this interaction are not clearly definable: Is a minimum threshold for each one essential? Is it possible for one to maximize if others are minimized? Are they dependent or exclusive,

complements or supplements? These are valid questions, but they are beyond the scope of our fundamental mission. The purpose of the Index is to reflect the economic and entrepreneurial environment in every country studied in as balanced a way as possible. The Index has never been designed specifically to explain economic growth or any other dependent variable; that is ably done by researchers elsewhere. The raw data for each component are provided so that others can study, weight, and integrate as they see fit." The data needed to compile the Index of Economic Freedom is itself obtained from other sources, such as, the World Economic Forum, World Competitiveness Report, World Justice Project, Rule of Law Index, Transparency International, Corruption Perceptions Index, TRACE International and the TRACE Bribery Risk Matrix, among others, as presented in the methodology section of the website (2). The twelve freedoms considered in constituting the Index of Economic Freedom are divided into four categories:

- 1. Rule of Law:** property rights, government integrity, judicial effectiveness
- 2. Government Size:** government spending, tax burden, fiscal health
- 3. Regulatory Efficiency:** business freedom, labor freedom, monetary freedom
- 4. Open Markets:** trade freedom, investment freedom, financial freedom

Definitions of the various freedoms can be found at the website (1), however a brief explanation of each freedom appears below:

1. *Property rights* are defined as the extent to which individuals can hold and utilize properties within a given country. A higher level of legal protection for property results in a higher index score. This was obtained by averaging survey data of physical, intellectual, and investor property rights, as well as the risk of expropriation and quality of land administration.

2. *Judicial effectiveness* measures the ability of a legal system to protect the rights of its citizens, including the enforcement of laws and appropriate punishment for breaking them. The index was obtained by averaging the factors of judicial independence, quality of the judicial process, and levels of favoritism within the judicial process.

3. *Government Integrity* examines levels of corruption within a given government, which can either increase or decrease public trust in the government. It thus also has an effect on economic activity. This index is obtained by averaging factors such as irregular payments/bribes, transparency in policy making, absence of corruption, perception of corruption, and governmental/civil transparency.

4. *Tax burden* measures tax rates on both personal and corporate income taxes as a percentage of GDP. The index is derived from

three sub-factors, including the top marginal tax rates on individual and corporate income, and the total tax burden as a percentage of GDP.

5. *Government spending* looks at the burden of all government expenditures. Though no ideal government expenditure has been identified, at a certain point it becomes restrictive towards economic growth. The index sets zero spending as the benchmark, which means the lesser developed countries may receive higher scores than more developed ones.

6. *Fiscal Health* looks at the debt burden and overall economic stability of a nation. The index is calculated by combining average deficits as percentage of GDP within three years (80%) with debt as a percentage of GDP (20%).

7. *Business freedom* measures the degree to which regulation can restrict the efficient operation of businesses. Business freedom is determined from thirteen factors, including the number of procedures, time, cost, and minimum capital for starting a business, and also the number of procedures, time, and cost of obtaining a license, closing a business, and getting electricity.

8. *Labor freedom* describes various aspects within the framework of a country's labor market. It consists of seven sub-factors, including the ratio of minimum wage to value added per worker, hindrance to hiring, rigidity of hours, difficulty of firing appropriate

employees, notice period, mandatory pay rate, and labor force participation rate.

9. *Monetary freedom* measures inflation and governmental actions that change prices. This index combines the weighted average inflation rate in the past three years and the extent to which the government manipulated prices via direct control/subsidies (qualitative).

10. *Trade freedom* measures the degree that tariff/non-tariff barriers affect the import/export of goods and services. This index combines a weighted tariff rate and a qualitative evaluation of non-tariff barriers.

11. *Investment freedom* refers to lack of restrictions on the flow of investment capital. Ideally industries would be able to move resources domestically and internationally with no restriction. Each country started with a score

of 100, and points were subtracted depending on restrictions in national treatment of foreign investment, foreign investment codes, restrictions on land ownership, sectoral investment restrictions, expropriation of investment without appropriate compensation, and controls on foreign exchange and capital (scores less than 100 were set to 0).

12. *Financial freedom* measures the efficiency of banks and freedom from government interference in the financial sector. Financial freedom combines five areas, including the extent of government regulation of financial services, the extent of state intervention in banks, governmental influence in allocation of credit, the extent of financial/capital market development, and openness to foreign competition.

Model development

In this section, some preliminary definitions are presented that are useful for the PCA setup (3). The expectation $E(X)$ is the average of the data. It measures the center of the data. The variance, $Var(X)$, measures the spread of the data around its mean.

$$A = (A_{11} A_{12} A_{21} A_{22}), A^T = (A_{11} A_{21} A_{12} A_{22})$$

The product an $n \times p$ matrix A and a $p \times q$ matrix B (the number of columns of the first and rows of the second must be the same) will result in a

Matrix Algebra

A transpose of a matrix A is denoted by A^T , and reflects a matrix across its main diagonal, or in other words, switches its row and column indices.

$n \times q$ matrix C where $c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$, $i = 1, \dots, m$ and $j = 1, \dots, p$

$$A = (A_{11} A_{12} A_{21} A_{22}), B = (B_{11} B_{12} B_{21} B_{22})$$

$$AB = (A_{11} B_{11} + A_{12} B_{21} \quad A_{11} B_{12} + A_{12} B_{22} \quad A_{21} B_{11} + A_{22} B_{21} \quad A_{21} B_{12} + A_{22} B_{22})$$

The eigenvector, also known as the characteristic vector, is a vector that scales a matrix by a value, known as the eigenvalue. In other words, the eigenvector (v) and eigenvalue(λ) satisfy $Av = \lambda v$ for a square matrix A

Model setup

A Principal Component Analysis (PCA) offers alternative ways to examine and reduce the data. Instead of calculating the average of the twelve different columns, PCA reduces the data into a two dimensional space while maximizing variance preservation and thus better illustrates the economic freedom relationship among all the countries. Additionally, with this two-dimensional reduction, it is possible to draw a clearer inference on how each of the countries perform on a specific freedom index (for example, how does the U.S perform on property rights compared to other countries?).

$$\operatorname{argmin}_{\Lambda, V \in O(p)} \sum_{i=1}^n (X_i - \Lambda_i V^T)^T (X_i - \Lambda_i V^T) = \operatorname{argmin}_{\Lambda, V \in O(p)} L(\Lambda_1 \dots \Lambda_n, V)$$

Where X_i represents the i -th row of the original X matrix,

Λ_i represents the i -th row of the reduced Λ matrix

Given the data matrix X as an input, the PCA attempts to find two additional matrices Λ, V such that the following equation holds:

$$X \approx \Lambda V^T$$

The reduction thus comes from the two additional matrices Λ, V , whose dimensions are lower compared to the original X matrix.

In this study, the X is the twelve indexes matrix with dimensions 180×12 and the Λ matrix has the dimensions of 180×2 . After the data reduction, the 180 different countries can be represented using the two dimensions of Λ matrix. However, to find this Λ, V matrix, the PCA implies the following two reduction criteria.

Minimizing the Difference Solution

The first method to find the matrix is through the following optimization problem:

V is an orthogonal matrix with $O(p)$ indicating $V^T V = I_p$

The solution to the above problem has the interpretation that the difference between X and ΛV^T are minimized and thus approximately satisfies:

$$X \approx \Lambda V^T$$

The optimization contains two parameters and we could derive the solution by using calculus:

$$\frac{dL(\Lambda_1 \dots \Lambda_n, V)}{d\Lambda_1} = \frac{\sum_{i=1}^n d[(X_i - \Lambda_i V^T)^T (X_i - \Lambda_i V^T)]}{\sum_{i=1}^n d[X_i^T X_i - \Lambda_i V^T X_i - X_i^T V \Lambda_i + \Lambda_i V^T V \Lambda_i]} \quad \square$$

$$\frac{\sum_{i=1}^n d[X_i^T X_i - \Lambda_i V^T X_i - X_i^T V \Lambda_i + \Lambda_i \Lambda_i^T]}{\square}$$

$$-2 X_i V^T + 2 \Lambda_i$$

Setting $\frac{dL(\Lambda_1 \dots \Lambda_n, V)}{d\Lambda_i} = 0$, we obtain:

$$\Lambda_i = X_i V^T$$

Substitute the solution back to $\argmin_{\Lambda, V \in O(p)} \sum_{i=1}^n (X_i - \Lambda_i V^T)^T (X_i - \Lambda_i V^T)$, we find:

$$\argmin_{V \in O(p)} \sum_{i=1}^n (X_i - X_i^T V V^T)^T (X_i - X_i^T V V^T)$$

Because $V^T V$ still satisfies the orthogonal constraint, we have:

$$V = \argmin_{V \in O(p)} \sum_{i=1}^n (X_i - X_i^T V)^T (X_i - X_i^T V)$$

$$V = \argmax_{V \in O(p)} \sum_{i=1}^n (X_i^T V)^T (X_i^T V)$$

The minimization thus reduces to a maximization problem, which implies the second model perspective.

Maximizing the Variance Solution

The second way to interpret the PCA is through the maximization of variance perspective.

Following the argument above, we have

$$V = \argmax_{V \in O(p)} \sum_{i=1}^n (X_i^T V)^T (X_i^T V)$$

When we have the data to be centered, we have:

$$\text{Var}(X) = E[X^2] - E^2[X] = E[X^2]$$

Comparing to the solution of matrix V , we combined with the first order condition implied came to the conclusion that we hope to find

matrix V such that the variance of $X_i^T V$ is maximized for all the i . This argument, that, when the data is centered, the question is

by $\frac{dL(\Lambda_1 \dots \Lambda_n, V)}{d\Lambda_1} = 0$ or $\Lambda_i = X_i V^T$, suggests

equivalent to finding λ_i such that the variance $XV = \Lambda Vi$

of the reduced dimension $\sum_{i=1}^n \text{Var}(\lambda_i)$ is maximized.

The solution to the maximization problem in matrix form is the eigen decomposition:

Where V is the eigenvector of the matrix X , Λ is the diagonal of the eigenvalue of matrix X . For this study, the program 'R' (supplementary file) was used to solve the PCA.

Results

As mentioned earlier, the projected principal (freedoms), based upon which, the performance components visualize the data according to of all the countries on specific economic their covariance matrix. The principal freedoms can be visualized. For simplicity, the components hence provide a more accurate R *prcomp* function was used to compute the relative number for the The Index of Economic principal components projection and the Freedom, than that calculated using the average *ggbiplot* function to visualize the projected of the twelve economic freedoms. In addition, principal component plots. (The R program is the PCA result also provides the orthogonal in the supplementary file). direction on each of the twelve columns

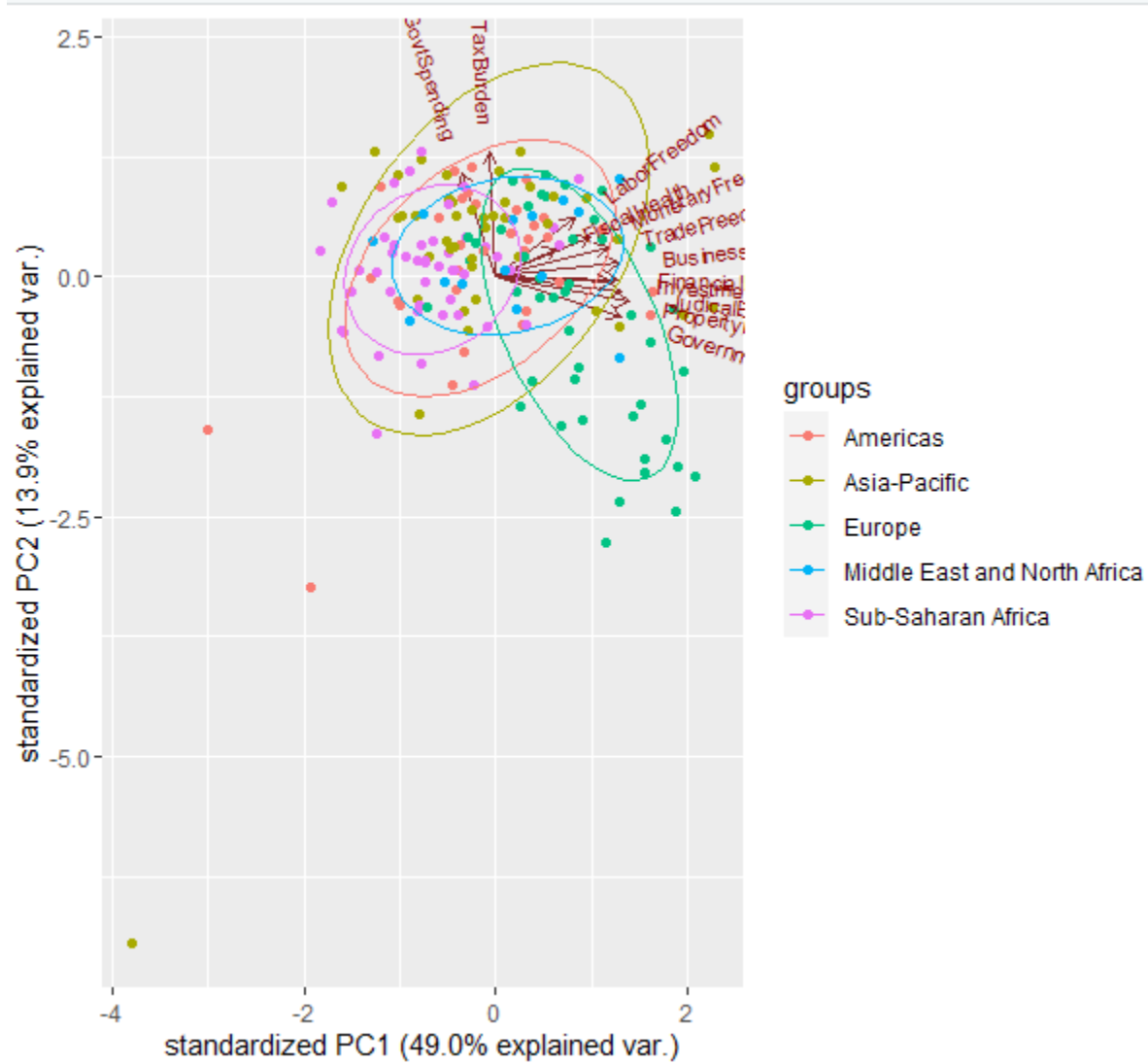


Figure 1: A plot of the first two principal components of each of the twelve economic freedoms.

Figure 1A (America Subplot)

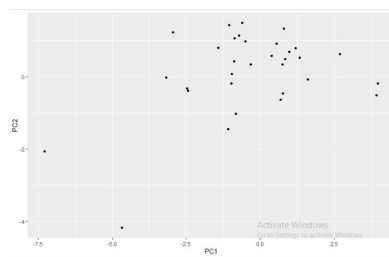


Figure 1B (Asia-Pacific Subplot)

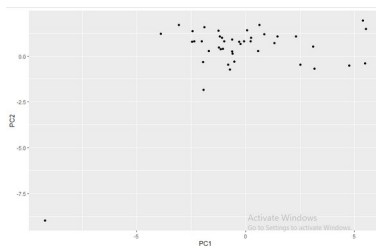


Figure 1C (Europe Subplot)

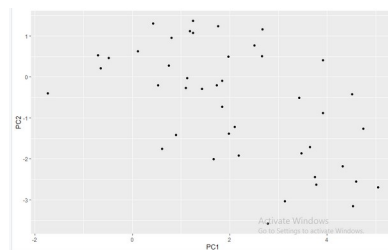


Figure 1D (Middle East and North Africa Subplot)

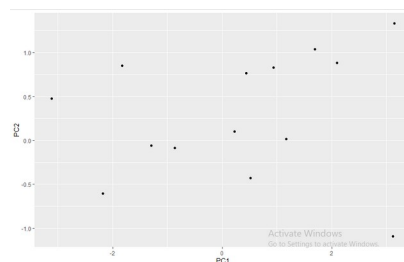
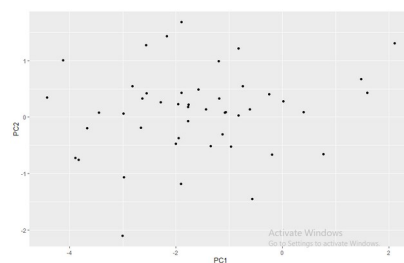


Figure 1E (Sub-Saharan Africa Subplot)



	Property Rights	Judicial Effectiveness	Government Integrity	Tax Burden	Govt Spending	Fiscal Health	Business Freedom	Labor Freedom	Monetary Freedom	Trade Freedom	Investment Freedom	Financial Freedom	Tariff Rate (%)	Income Tax Rate (%)	Corporate Tax Rate (%)	Tax Burden % of GDP	Govt Expenditure % of GDP
Property Rights		0.8173952	0.8508033	-0.15001751	-0.23427826	0.34689403	0.77214287	0.42770537	0.47170817	0.6388295	0.661325923	0.703915625	-0.494782393	0.15350977	-0.235862249	0.475432913	0.097938274
Judicial Effectiveness			0.87392915	-0.118770362	-0.134274947	0.333235913	0.72244253	0.45645715	0.43602025	0.553013006	0.546532243	0.53585523	-0.438075434	0.178508273	-0.219552636	0.348527272	0.032395528
Government Integrity				-0.215458734	-0.239157228	0.30202355	0.75288786	0.40727254	0.41018424	0.536788034	0.578495541	0.613740636	-0.397751405	0.221766379	-0.249039821	0.415825389	0.16549036
Tax Burden					0.395463315	0.015300506	0.091534071	0.257959311	0.12402063	0.174641243	-0.14614678	-0.05150184	-0.263073623	-0.801257849	-0.435913416	-0.527530789	-0.34702941
Govt Spending						0.08244739	-0.143255481	0.055157342	-0.010144485	-0.127177304	-0.0853523	-0.127770685	0.076464938	-0.073857802	0.211174638	-0.62775543	-0.883423417
Fiscal Health							0.339322967	0.180240593	0.315524201	0.305286105	0.272897779	0.253705212	-0.262968041	0.073795553	-0.062364716	0.149353229	-0.139453384
Business Freedom								0.507780235	0.477335776	0.645153912	0.585746816	0.607337838	-0.54501281	0.000328184	-0.325386248	0.386557228	0.00169534
Labor Freedom									0.35482853	0.394455893	0.290354409	0.328435478	-0.358332394	-0.15961935	-0.163990871	0.08325458	-0.107832894
Monetary Freedom										0.44747239	0.595260842	0.430284414	-0.500273032	0.077865266	-0.174784406	0.24966037	-0.12189534
Trade Freedom											0.593303578	0.627765621	-0.949155156	-0.024923402	-0.300270024	0.33942711	-0.009524303
Investment Freedom												0.788393370	-0.448633622	0.161775586	-0.15351591	0.436662632	-0.05367755
Financial Freedom														0.07789103	-0.253573461	0.380253656	0.030816701
Tariff Rate (%)															0.253589198	-0.285788732	0.072338905
Income Tax Rate (%)																0.557425363	0.342893965
Corporate Tax Rate (%)																	0.09553815
Tax Burden % of GDP																	0.462478353
Govt Expenditure % of GDP																	

Figure 2 (Standardized Covariance Table)

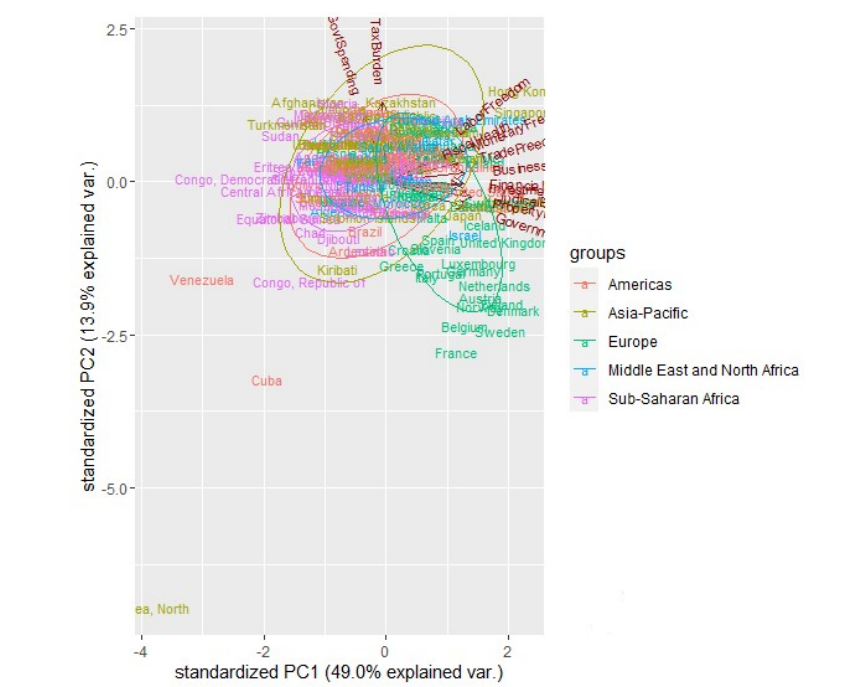


Figure 3A (Full plot with country labels)

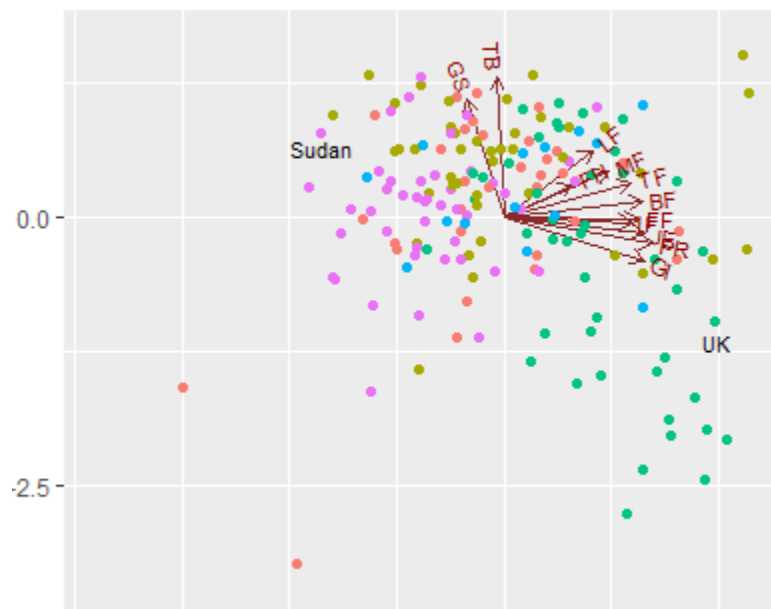


Figure 3B: Full plot with vectors (Economic freedoms appear as self-evident abbreviations)

	[,1]	[,2]
[1,]	-0.34668649	-0.16197699
[2,]	-0.32725763	-0.10787208
[3,]	-0.32606020	-0.23528575
[4,]	0.00301645	0.63878958
[5,]	0.06942010	0.56345783
[6,]	-0.17027138	0.15720895
[7,]	-0.32623840	0.03692613
[8,]	-0.21350214	0.27734694
[9,]	-0.24883221	0.17677427
[10,]	-0.29965332	0.12012828
[11,]	-0.30232133	-0.05625906
[12,]	-0.31690285	-0.04200017

Figure 3C: Eigenvectors for the twelve economic freedoms obtained from the R program (supplementary file), Row labels are the same as those in Figure 2.

Region-wise analysis

Figure 1 shows that, in general, European countries' social welfare programs that require large amounts of money to support. In addition, government spending and tax burden, but score well in the other ten economic freedoms, while the Middle East tends to be around average for all the freedoms. Europe's poor performance with regard to tax burden and government spending can be attributed to many European countries' social welfare programs that require large amounts of money to support. In addition, government spending is not weighted based on GDP, many large, wealthy countries such as the US, UK, and Germany perform poorly on this freedom. Higher government spending on social welfare programs generally

also correlates with higher tax rates, as it falls on the citizens of the nation to support higher spending, explaining why tax burden and government spending are closely correlated. The other ten economic freedoms are grouped together (in the sense that a country that performs well on any one freedom will likely perform well on the other nine), largely due to consistency within a particular country's legal and regulatory framework. For example, a country with insignificant government corruption is more likely to have a just judicial

system, and a country with higher monetary freedom is also more likely to have higher freedom in the other economic aspects. If the countries are clustered according to their overall performance on the twelve economic freedoms, the European countries tend to perform similar to each other. The Asian countries, on the other hand, tend to demonstrate a larger variability. These results are in general agreement with the map plot (Figure 4) from the Heritage website.

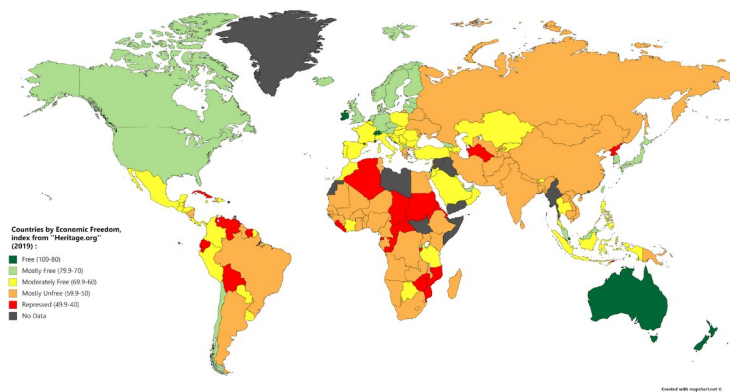


Figure 4: Country-wise analysis from the Heritage website

To further verify the conclusions from Figure 1, the economies of specific countries can be examined to see if the results are consistent with those presented in Figure 1. Background research on the economy of selected countries' was performed using the Journal of Economics to obtain relevant data (4).

United Kingdom Economy

Based on Figure 3A, the United Kingdom performs well in labor freedom, fiscal health,

monetary freedom, trade freedom, business freedom, financial freedom, investment freedom, judicial effectiveness, property rights, and governmental freedom, but scores poorly in terms of government spending and tax burden. These results also apply when compared to other European countries, which can be seen in Figures 1 and 4. Similar to many European nations, the UK has many state-run social programs such as social welfare, universal basic income, and free healthcare.

These programs contribute to the high government spending in the UK, as well as significant expenditure in education and defense. It is also generally considered that public-sector corruption within the UK is low, the judicial system is mostly effective at identifying and charging criminals, and that property rights in the UK allow citizens to obtain and maintain property without an onerous process. These observations are consistent with the visual conclusions depicted in Figure 3A. A standard minimum wage and worker protection provides high levels of labor freedom. Finally, the British economy is considered to be a well-developed, capitalistic, market driven economic system. It is also one of the largest importing and exporting countries in the world. These factors require a system to have high levels of trade, monetary, and investment freedom, and lead to a high level of fiscal health. Overall, the information derived from Figure 3A regarding the United Kingdom is largely consistent with background research on various aspects of British economic freedom (5).

Sudan Economy

Another example that could be examined is the country of Sudan. Sudan performs well on government spending, which is slightly misleading due to the way the index defines government spending. Sudan's government spending is largely due to the lack of money to

spend, due to the widespread poverty within the country. Sudan's taxes are also quite low compared to much of the developed world, largely due to many of its citizens relying on subsistence agriculture. In rural Sudan, insecurity regarding land ownership results from corruption and instability within the government. As such, Sudan performs poorly with regard to property rights and government integrity. Sudan also imposes tariffs on many goods that compete with locally produced goods, and there are often many barriers to export to Sudan, leading to a low trade freedom score. Corruption and instability within Sudan's government contribute to the poor scores of many freedoms such as monetary, investment, labor, and business freedom, as the legal framework is both not well defined and still largely flawed. This instability also leads to significantly decreased judicial effectiveness, as a government that cannot hold itself together cannot hope to manage the rights of its citizens. These factors lead to an overall poor fiscal health for Sudan, which is consistent with the conclusions from Figure 3A. Even when compared to other African countries in Figure 4, Sudan performs poorly on the economic freedoms, which is a result of widespread corruption. This is largely a lingering effect of the 2011 Sudanese civil war (6), whose effects can be clearly seen in it's Index of Economic Freedom.

Reduction of Attributes

Using PCA, two principal components could capture or explain the variance in 62.9%, or $\sim 2/3^{\text{rd}}$ of the data. The dimensionality of the twelve attributes was thus reduced to two. Additionally, the PCA showed that ten of the economic freedoms were closely correlated, as were the remaining two. Thus, it was reasonable to mimic the Index of Economic Freedom using only two of these twelve economic freedoms, one selected from each group. An examination of the eigenvectors on the PCA revealed that the business freedom (row 4 of Figure 3C) and tax burden Row 7 of

Figure 3C) appeared to be approximately perpendicular (83.8°), hence setting them as the x/y axes would yield a similar result to the PCA. Examining the general locations of each region in Figure 3A, it appeared that using only these two freedoms could be used to generate an approximation for the Index. This meant that it would be possible to generate a relatively accurate picture of the Index of Economic Freedom by measuring only two freedoms, which obviously has practical applications in areas where measuring other attributes may be difficult.

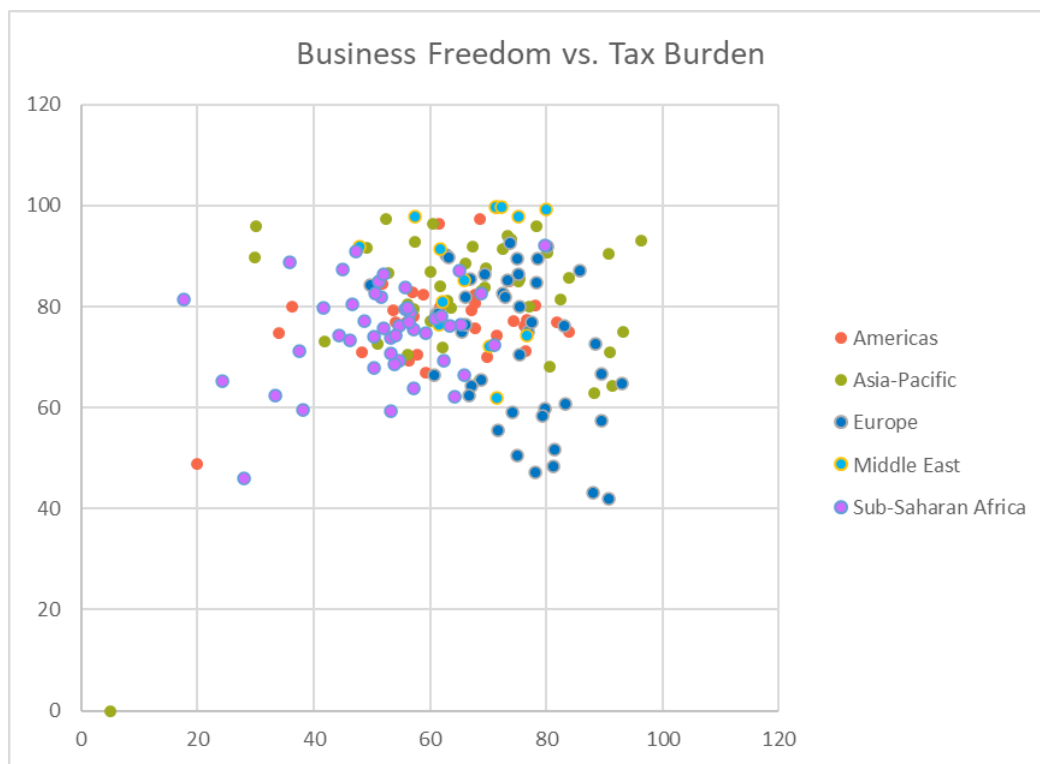


Figure 5: Two-index approximation of the Index of Economic Freedom

PCA was also performed using an online tool variance scaling for rows and SVD with (7), in turn derived from the work of Metsalu imputation for the PCA method. The other et.al. (8). The pre-processing options were unit

parameters were set to default. Figures 6 and 7 present the results of this analysis.

Figure 6: Variance explained by principal components using the online PCA tool.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12
Individual	0.47	0.19	0.11	0.07	0.04	0.03	0.02	0.02	0.02	0.01	0.01	0.00
Cumulative	0.47	0.67	0.77	0.84	0.88	0.91	0.94	0.96	0.97	0.99	1.00	1.00

Figure 7: Correlation with the original variables with the Principal components

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12
property rights	-7.32	1.64	1.82	2.04	-2.39	0.57	-0.79	0.31	-1.03	0.60	3.81	0.00
government integrity	-13.71	0.86	-0.72	2.14	-1.23	0.50	0.23	-0.06	-2.54	2.55	-2.22	-0.00
judicial effectiveness	-11.54	-2.60	-0.97	3.48	-0.42	-1.03	0.04	-3.61	1.67	-2.48	-0.43	0.00
tax burden	13.15	-3.57	3.86	0.06	2.35	3.42	-1.71	-3.03	-0.33	1.25	-0.02	0.00
government spending	10.38	-11.40	-7.35	-0.67	-3.60	-0.72	0.10	0.68	-0.53	-0.06	0.01	-0.00
fiscal health	9.06	13.56	-7.93	1.84	0.70	0.21	-0.26	-0.32	0.32	-0.04	-0.01	-0.00
business freedom	0.45	-0.04	3.47	2.60	-1.64	0.93	-1.35	2.78	4.05	0.83	-0.86	0.00
labor freedom	-0.32	-4.47	-0.27	3.47	6.36	-2.29	-0.35	2.26	-1.00	-0.51	0.33	0.00
monetary freedom	8.27	1.99	5.08	-0.57	-0.70	-3.48	4.41	-0.91	0.41	1.05	0.13	0.00
trade freedom	4.62	3.23	5.05	-0.88	-1.92	1.60	0.03	1.59	-2.33	-3.23	-1.00	0.00
investment freedom	-3.89	2.06	0.78	-7.78	0.22	-3.34	-3.36	-0.41	0.21	0.25	-0.17	-0.00
financial freedom	-9.15	-1.24	-2.80	-5.73	2.27	3.62	3.01	0.72	1.12	-0.22	0.44	-0.00

Table 1: Orthogonality search among the freedom categories for Principal components 1 and 2

	PC1	PC2
Rule of Law (Property rights + Government integrity + Judicial effectiveness)	-10.86	-0.033
Government size (Government spending + Tax burden + Fiscal health)	+10.86	-5.86
Regulatory effectiveness (Business freedom + Labor freedom + monetary freedom)	+2.8	-0.84
Open markets (Trade freedom + Investment freedom + Financial freedom)	-2.81	+1.35

The principal components do not represent the original variables; rather some linear combinations of those variables. One of the objectives of the study was to reduce dimensionality so that any two of the original variables could be plotted against each other with the maximum variance capture (or minimum data loss). This meant identifying those variables in PC1 and PC2 that showed maximum orthogonality. The data from Figure 7 was hence processed into four categories for the first two Principal components and is presented in Table 1. It can be seen that maximum orthogonality is obtained when

1. the variables included in the Government size category in PC1 are plotted against the original variables included in the Rule of Law category in PC2 or
2. the variables included in the Rule of Law or Government size categories in PC1 are plotted against the variables included in the Regulatory effectiveness category in PC2.

Tables 2 and 3 present the correlation of the variables from the orthogonal categories obtained from the covariance matrix. It follows that the lesser the correlation between the original variables, the better those two variables are in capturing all the variance in the entire data set.

Table 2: Correlation between the variables included in Government size and those included in the Rule of Law

	Property rights	Government integrity	Judicial effectiveness
Government spending	-0.23	-0.3	-0.13
Tax burden	+0.48	+0.42	+0.35
Fiscal health	+0.35	+0.31	+0.33

Table 3: Correlation between the variables included in the Rule of law and Government size categories versus those included in the Regulatory effectiveness category

	Property rights	Government integrity	Judicial effectiveness	Government spending	Tax burden	Fiscal health
Business freedom	+0.77	+0.72	+0.72	-0.14	+0.09	+0.34
Labor freedom	+0.42	+0.41	+0.46	+0.06	+0.25	+0.18
Monetary freedom	+0.47	+0.41	+0.44	-0.01	+0.26	+0.32

It can be seen from Tables 2 and 3 that when Monetary freedom is plotted against Government spending, or Labor freedom against Government spending, or Business freedom against Tax burden, the model can capture the most variance in the original data. However, Figure 7 shows that the individual variables of Monetary freedom from PC1 and Government spending from PC2 are inversely correlated (not orthogonal), as are Government

spending from PC1 and Labor freedom from determined that the Index of Economic PC2. Orthogonality is best maintained between Freedom could be adequately represented by Business freedom (PC1 or PC2) and Tax two economic freedoms; Tax burden and burden (PC2 or PC1). Hence, the result from Business freedom. An accurate representation the eigenvector values from the PCA (Figure of the Index of Economic Freedom, as a 3C) is supported by the online PCA tool and relative barometer of useful, productive and the covariance matrix. beneficial economic activity for 182 countries

can be obtained using the these two economic

An examination of the eigenvectors of the freedoms.

covariance matrix and the orthogonality maximization from the online PCA tool

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