Peer-Review

Marotta, Pedro. 2026. "A Scaling-Parameter Framework for Perimeter and Area in Self-Similar Planar Fractals." *Journal of High School Science* 10 (2): 275–95. https://doi.org/10.64336/001c.162173.

This paper is more of a pedagogical nature than of an original contribution. I have revised it into an original contribution (which is publishable). Please review the attached chatgpt files (make sure you agree with the mathematical model and explanation) and reframe your title, theme, premise and analysis accordingly. Please revise your manuscript accordingly. Below is a brief summary of how your original paper has changed:

This revised work advances beyond the original paper by shifting from a descriptive analysis of individual fractals to a unified, theory-driven framework. Rather than treating examples such as the Koch snowflake and Sierpinski constructions in isolation, the paper now formulates a general scaling model parameterized by (N) and (r), identifies the precise regime ($r < N < r^2$) governing the coexistence of infinite perimeter and finite area, and interprets this regime as a critical balance between boundary amplification and geometric saturation. In addition, the introduction of a phase diagram in ((N,r))-space provides a systematic classification of geometric behavior, while the integration of physical concepts from diffusion-limited growth offers a mechanistic explanation for why such structures arise. Together, these changes transform the work from an expository exploration into a cohesive framework that unifies classical results and provides broader conceptual insight.

Response to Reviewers — Manuscript 3136520

Manuscript title (revised): *A Scaling-Parameter Framework for Perimeter and Area in Self-Similar Planar Fractals*

Original title: *Could we create a fractal with an infinite perimeter and a finite area by studying existing ones?*

Author: Pedro Marotta

Date of resubmission: 2026-04-28

The author thanks the editor and both reviewers for the careful and constructive review of the original manuscript. The revised manuscript adopts the structural direction suggested by Reviewer 1 (organizing the paper around a two-parameter (N, r) framework with a regime classification) and addresses every point raised by both reviewers as detailed below. Each reviewer comment is reproduced verbatim, followed by a description of the response and the specific section or paragraph of the revised manuscript in which the response is implemented. Where suggestions in the attached ChatGPT files were not adopted, the reasons are stated explicitly and respectfully.

Reviewer 1

R1. Comment (verbatim)

"This paper is more of a pedagogical nature than of an original contribution. I have revised it into an original contribution (which is publishable). Please review the attached chatgpt files (make sure you agree with the mathematical model and explanation) and reframe your title, theme, premise and analysis accordingly. Please revise your manuscript accordingly. Below is a brief summary of how your original paper has changed:

This revised work advances beyond the original paper by shifting from a descriptive analysis of individual fractals to a unified, theory-driven framework. Rather than treating examples such as the Koch snowflake and Sierpinski constructions in isolation, the paper now formulates a general scaling model parameterized by (N) and (r), identifies the precise regime ($r < N < r^2$) governing the coexistence of infinite perimeter and finite area, and interprets this regime as a critical balance between boundary amplification and geometric saturation. In addition, the introduction of a phase diagram in ((N,r))-space provides a systematic classification of geometric behavior, while the integration of physical concepts from diffusion-limited growth offers a mechanistic explanation for why such structures arise. Together, these changes transform the work from an expository exploration into a cohesive framework that unifies classical results and provides broader conceptual insight."

Response (organized by sub-point)

The reviewer's comment encompasses several substantive directions. Each is addressed individually below.

R1.a — Reframing the paper around a unified (N, r) framework rather than treating each fractal in isolation.

This direction has been adopted. The revised manuscript is organized around an explicit two-parameter framework (N = number of self-similar pieces per iteration; r = inverse linear scale factor) and presents the four worked fractals as instances of the framework rather than as isolated examples. The framework is introduced in Section 2 (Preliminaries), the worked examples in Section 3 are uniformly described in (N, r) terms, and Section 4 contains the unified scaling laws (eq22)–(eq24) and the regime classification.

Location in revised manuscript: §2.1, §2.3, §3.1–§3.4, §4.1.

R1.b — Title and theme reframed accordingly.

The title has been changed to *"A Scaling-Parameter Framework for Perimeter and Area in Self-Similar Planar Fractals"*, and the introduction (Section 1) has been rewritten to open with the phenomenon (the Koch snowflake's infinite perimeter and finite enclosed area), the question (under what general conditions does this behavior occur), and the framework as the paper's organizing contribution. Personal-narrative material from the original manuscript (the dice-game introduction, the "amazed by chaos" prose, the first-person evaluation) has been removed in keeping with R2.5 and JHSS guidelines on superlative or opinionated language.

Location in revised manuscript: title page; §1 throughout; §7 (Conclusion).

R1.c — Identification of the regime $r < N < r^2$ as governing the coexistence of infinite perimeter and finite area.

The condition $r < N < r^2$ is now treated explicitly in the revised manuscript and is correctly identified as the regime in which the iteration-n perimeter (under the perimeter conventions of §2.4) diverges. However, the revised manuscript presents this condition as a corollary of the similarity-dimension formula $D = \log(N)/\log(r)$, since taking logarithms of the bounds yields $1 < D < 2$ — the standard textbook condition for non-integer planar fractal dimension (3, 4). It is therefore presented as a unifying corollary, not as a new theorem.

A stronger claim of the form "$r < N < r^2$ is necessary and sufficient for both infinite perimeter and finite area" was considered but not adopted. Two of the four worked examples — the Sierpinski triangle (N = 3, r = 2) and the Sierpinski carpet (N = 8, r = 3) — satisfy $r < N < r^2$ yet have asymptotic two-dimensional Lebesgue measure equal to zero rather than positive finite. By Hutchinson's theorem (5), any self-similar set with similarity dimension $D < 2$ has zero two-dimensional Lebesgue measure as a set, so the Sierpinski examples cannot be characterized as having "finite (positive) area" within the regime. To address this, the revised manuscript distinguishes between additive constructions (Koch-type, where the object of interest is the region bounded by the iterated curve) and subtractive constructions (Sierpinski-type, where the object of interest is the iterated set itself); within the intermediate-dimension regime, additive constructions yield positive finite enclosed area under a stated non-overlap assumption, while subtractive constructions yield zero area.

This refinement is presented as the small pedagogical contribution of the paper. It is asserted within the construction classes specified in §2.2 and is not claimed as a universal property of all self-similar planar fractals.

Location in revised manuscript: §4.3 (regime classification, with explicit derivation that $r < N < r^2 \Leftrightarrow 1 < D < 2$); §4.4 (construction-class refinement); §6.1 and §6.2 (scope statements).

R1.d — Phase diagram in (N, r)-space.

Figure 5 of the revised manuscript plots the (N, r) parameter space with three regimes shaded — subcritical ($N \leq r$), intermediate-dimension ($r < N < r^2$), and supercritical ($N \geq r^2$) — and the four

worked examples marked. The figure is labeled as a parameter-space regime plot rather than a phase diagram in the thermodynamic sense, since no thermodynamic phase transition is being claimed; this language choice is consistent with R2.4 (avoidance of overreaching conclusions).
Location in revised manuscript: Figure 5; §5; figure-legends file.

R1.e — Interpretation of the intermediate-dimension regime as a "critical balance between boundary amplification and geometric saturation."

The revised manuscript states the asymptotic behavior of the perimeter and area scaling laws explicitly (§4.2) and shows that the intermediate-dimension regime is the parameter band in which perimeter diverges while area, in the additive case, remains bounded under non-overlap. This is the literal mathematical content of the suggested "critical balance" interpretation. Stronger language — "criticality," "instability versus saturation," "phase-like boundary" — has been used sparingly to avoid attributing thermodynamic or non-equilibrium-physics meaning to a purely geometric scaling result, in keeping with R2.4. The mathematical substance of the suggested interpretation is preserved; the physical-criticality framing is not adopted.
Location in revised manuscript: §4.2; §4.3; §4.4.

R1.f — Integration of physical concepts from diffusion-limited growth as a mechanistic explanation.

This direction has not been incorporated as a substantive part of the paper's contribution. Diffusion-limited aggregation (8) and related stochastic growth models produce fractal limit sets via random rather than deterministic iteration rules; their (non-integer) fractal dimensions arise from stochastic clustering and are computed from simulation or scaling theory rather than from the deterministic similarity-dimension formula $D = \log(N)/\log(r)$ used in this paper. Importing language from stochastic-fractal physics without importing its mathematics would risk an overreach beyond what the deterministic framework presented here can support, contrary to R2.4. Connections between deterministic self-similar IFS fractals and stochastic growth processes are real and well-studied in the research literature (see, e.g., the discussion of physical fractal models in (3, Chapter 16)), but a substantive treatment is beyond the scope of a high-school-level pedagogical paper. The Discussion (Section 6.2) acknowledges the existence of stochastic fractal models, situates them as belonging to a different mathematical class, and explicitly states that this paper makes no claims about them.
Location in revised manuscript: §6.2.

R1.g — Reconciliation with the four attached ChatGPT files.

The four attached files provided detailed mathematical and structural suggestions. The author has reviewed each and has adopted the following points: (i) the (N, r) parameterization (Files 1, 4); (ii) the perimeter and area scaling forms (N/r)^n and ratio N/r^2 (File 1); (iii) the three-regime classification (Files 1, 4); (iv) a parameter-space plot showing the regimes and worked examples (File 4); (v) a reorganized title, abstract, introduction, and conclusion in scientific register (File 4). The following points from the attached files have not been adopted, with reasons:

- The "Main Result" labelled "$1 < N/r < r$" or "$r < N < r^2$ iff infinite perimeter AND finite area" (File 1) is presented in the revised manuscript as a corollary of the dimension formula rather than a theorem, for the reasons given in R1.c above.
- The optimization derivation yielding an "optimal N/r" (File 3) has not been incorporated. The derivation in File 3 starts from the functional $E = P/A - \lambda P$ and produces an optimum $x^* = N/r = r \cdot \log(\ell) \cdot (1-\lambda) / (\log(\ell) - \log(r))$. Two issues prevented adoption: first, the functional E mixes quantities of different dimensional units (P has units of length, A of area, so P/A has units of inverse length, while λP has units determined by λ; no unit specification is given). Second, in the fine-scale limit $\ell \to 0$ the expression for x^* simplifies to $r(1-\lambda)$, which lies on the boundary of the admissible region $1 < x^* < r$ rather than in its interior; an interior optimum cannot be claimed. The author is unable to defend the resulting expression and has therefore omitted this material rather than risk including a derivation that could not withstand re-review. A simpler regime classification has been used instead.
- The "diffusion-limited growth" mechanistic explanation in Files 1 and 4 has been treated as in R1.f above.

- The “transport efficiency” and “natural systems” interpretations in File 2 have not been adopted, as they pertain to physical systems (branching networks, biological structures) outside the scope of this paper’s deterministic IFS framework.

The author’s verification of each formula in the attached files is acknowledged in Acknowledgments and is included in the editorial-disclosure statement therein.

Location in revised manuscript: Acknowledgments (statement of AI assistance); §6.1, §6.2 (scope and limitations).

Reviewer 2

R2.1 Comment (verbatim)

“Please verify that all links to the references point to the correct source.”

Response

All references in the original manuscript that pointed to non-archival sources (Wikipedia, YouTube, Khan Academy, blog posts) have been replaced with primary or canonical academic sources where such replacement is possible while preserving the citation’s purpose. The revised reference list contains eight numbered references, each with either a DOI (where assigned) or an ISBN with publisher and location (where the source is a book that predates DOI assignment, e.g. Mandelbrot 1982). Each DOI in the revised reference list has been verified by the author to resolve to the correct source.

The substitution from Wikipedia/YouTube to academic sources is deliberate and is noted here: JHSS guidelines require a DOI after each reference where available; this requirement and the lower citation reliability of unpaginated web sources together motivated the substitution. Some figures in the original manuscript credited to web sources have been replaced by the author’s own renderings or by figures derived from the cited textbooks.

Location in revised manuscript: References (after Acknowledgments); throughout the body where citations occur.

R2.2 Comment (verbatim)

“The authors must disclose and acknowledge any assistance received in the preparation of this manuscript, including but not limited to editorial, technical, analytical, or writing support. All such contributions must be clearly stated in the Acknowledgments section.”

Response

An Acknowledgments section has been added immediately before the References section. The Acknowledgments state plainly that (i) Reviewer 1 of the original submission suggested the (N, r) parameterization that is adopted as the paper’s organizing framework in the revision; (ii) AI tools (specifically ChatGPT and Claude) were used during preparation of this revision for editorial assistance, structural suggestions, and checking of algebraic exposition; and (iii) all mathematical derivations and the Koch-style construction on a square presented in §3.4 are the author’s own work, with the author having verified all mathematical content and accepting full responsibility for the manuscript.

Location in revised manuscript: Acknowledgments section.

R2.3 Comment (verbatim)

“All assumptions (including implicit assumptions) must be explicitly stated and clearly justified. The authors should explain why each assumption is reasonable and discuss its impact on the results and conclusions.”

Response

The revised manuscript states all assumptions explicitly. The principal assumptions are:

(i) The construction is a deterministic self-similar iterated function system in the plane satisfying the open set condition (stated in §2.1, §2.3, with Hutchinson’s theorem cited for the equivalence of similarity and Hausdorff dimensions in §2.3 and §6.5).

(ii) The perimeter convention is stated explicitly per construction class in §2.4. The convention adopted for subtractive constructions (total edge length of all sub-pieces at iteration n) is acknowledged as non-standard; alternative conventions (convex hull boundary, Hausdorff one-measure) yield different values (§6.4).

(iii) The non-overlap assumption for additive constructions is stated explicitly in §3.4 (where it is needed for the area calculation of the Koch-style construction on the square) and discussed in §6.3. The numerical area value $A_\infty = 2$ is contingent on this assumption, which is verified visually for the first three iterations in Figure 4 but is not proved for arbitrary iteration depth in this paper. The contingency is recorded.

(iv) The two-class dichotomy (additive and subtractive) is asserted within the construction classes defined in §2.2, not as a universal property of all self-similar planar fractals (§4.4, §6.1).

The impact of each assumption on the results is discussed in the corresponding subsection of Section 6 (Discussion and limitations).

Location in revised manuscript: §2.1, §2.3, §2.4 (assumptions); §3.4 (non-overlap assumption explicit in derivation); §6.1–§6.5 (impact discussion).

R2.4 Comment (verbatim)

"Avoid overreaching conclusions that extend beyond what is supported by the data and analysis."

Response

The revised manuscript has been rewritten to avoid overreaching language. Specific changes:

- The condition $r < N < r^2$ is presented as a corollary of the similarity-dimension formula rather than as a theorem or main result (§4.3).
- The construction-class refinement is asserted within the construction classes defined in §2.2, not as a universal property of all self-similar planar fractals (§4.4, §6.1, §7).
- The parameter-space plot is labeled as a "regime figure" rather than a "phase diagram in the thermodynamic sense" (§5).
- Connections to physical-criticality frameworks (instability, saturation, diffusion-limited growth) are mentioned in the Discussion as belonging to a different mathematical class outside the scope of this paper, and no claim of equivalence or unification is made (§6.2).
- Superlative or opinion-bearing language from the original manuscript ("amazing," "fascinating," "I believe," "I think," "fundamentally," "fabulously") has been removed throughout. The Conclusion (§7) states the contribution honestly as pedagogical rather than novel in the research-mathematical sense.
- The Koch-style construction in §3.4 is attributed explicitly to the author (Pedro Marotta), who invented it as part of the original Extended Essay investigation. The iteration rule, the similarity-dimension calculation, the perimeter scaling, and the closed-form area derivation in §3.4 are the author's own work.

Location in revised manuscript: §3.4 (Koch-square framing); §4.3, §4.4, §5 (regime claims); §6.1, §6.2 (scope); §7 (Conclusion); throughout, copy-edit removing superlatives.

R2.5 Comment (verbatim)

"Verify that you have used past perfect tence and third person throughout the manuscript wherever applicable."

Response

The manuscript has been rewritten in the third person throughout. First-person constructions ("I will," "we will," "I was amazed," "I researched," "we get," "we are") have been removed or replaced with passive or neutral third-person formulations. The personal-narrative introduction (the dice-game in 8th grade math class) has been removed, as has the first-person Evaluation section at the end of the original. The Acknowledgments section is the only place where first-person reference to the author appears, and there only in the standard formal-acknowledgment register.

The dominant tense in the revised manuscript is the simple present (the standard tense for mathematical exposition) when describing definitions, derivations, and properties; the simple past is used for historical facts (e.g., "Sierpiński described this construction in 1915"); the past perfect appears where applicable (e.g., when describing what an earlier construction had established before a later one was introduced). The author has reviewed the manuscript for tense consistency and applies past perfect where context warrants it; the dominant register, however, is the present-tense expository style standard in mathematical writing rather than past perfect throughout, since most statements describe atemporal mathematical content.

Location in revised manuscript: throughout; §1 (rewritten introduction without dice-game narrative); §7 (rewritten conclusion without first-person evaluation); Acknowledgments (only first-person reference, in standard register).

Summary table — coverage of each comment

Comment	Section(s) of revised manuscript
R1.a (unified framework)	§2.1, §2.3, §3.1–§3.4, §4.1
R1.b (title/theme reframe)	Title page, §1, §7
R1.c (regime r < N < r^2)	§4.3, §4.4, §6.1, §6.2
R1.d (phase diagram)	§5, Figure 5, figure legends
R1.e (criticality interpretation)	§4.2, §4.3, §4.4
R1.f (DLA mechanistic explanation)	§6.2
R1.g (attached ChatGPT files reconciliation)	Acknowledgments, §6.1, §6.2
R2.1 (verify reference links)	References, throughout
R2.2 (disclose all assistance)	Acknowledgments
R2.3 (state and justify assumptions)	§2.1, §2.3, §2.4, §3.4, §6.1–§6.5
R2.4 (avoid overreach)	§3.4, §4.3, §4.4, §5, §6.1, §6.2, §7, throughout
R2.5 (third person, past perfect)	Throughout; §1, §7, Acknowledgments

The author thanks the editor and reviewers again for their careful attention. Any remaining concerns or requests for further clarification will be addressed promptly upon notification.

Pedro Marotta 2026-04-28

Thank you for addressing my comments. I agree also with your pushback on the physics diffusion angle since it does not fit cleanly within a purely mathematical framework. However, I have 2 remaining concerns.

1.You state that the you only visually verify the non-overlap assumption for the first 3 iterations. Since the key results of A_infinity=2 depends on this; can you explicitly state that the non-overlap assumption is hence an unproven yet clearly bounded claim? Or something along those lines?

2.I think you are underselling your contribution. While I agree that no new theorem, condition, regime discovery……is made; the framework does function as a diagnostic tool infer asymptotic perimeter and area behavior without re-deriving each example. Please also see the attached pdf chatgpt file for more defensible claims within the framework. I would like you to reframe your contribution precisely in terms of novelty in formulation and synthesis (and pedagogy). The chatgpt file also includes a rewrite of the abstract and conclusion after such reframing. You can rewrite in your own words and terms.

Response to Reviewer (Round 2) — Manuscript 3136520

Manuscript title: *A Scaling-Parameter Framework for Perimeter and Area in Self-Similar Planar Fractals*

Author: Pedro Marotta

Date of resubmission: 2026-05-04

The author thanks the editor (Shireesh Apte) and the reviewer for the additional comments on the round-1 revision. Each reviewer comment is reproduced verbatim below, followed by a description of the response and the specific section(s) of the round-2 revised manuscript in which the response is implemented.

Reviewer comment 1 (verbatim)

"Thank you for addressing my comments. I agree with your pushback on the physics diffusion angle since it does not fit cleanly within a purely mathematical framework."

Response

The author thanks the reviewer for accepting the prior pushback on the diffusion-limited / non-equilibrium-physics framing. No further action is required on this point. The position taken in the round-1 revision (§6.2 of the manuscript and R1.f / R1.g of the round-1 response) is retained

unchanged: stochastic fractal models lie outside the scope of the deterministic IFS framework presented here and are acknowledged but not analyzed.
Location in round-2 revised manuscript: §6.2 (unchanged from round 1).

Reviewer comment 2 (verbatim)

"You state that the you only visually verify the non-overlap assumption for the first 3 iterations. Since the key results of A_infinity=2 depends on this; can you explicitly state that the non-overlap assumption is hence an unproven yet clearly bounded claim? Or something along those lines?"

Response

This comment has been adopted in full. The status of the value $A_\infty = 2$ in (eq21) is now stated explicitly, in two places, as an *unproven yet clearly bounded conditional claim*, using the language the reviewer suggests.
In §3.4, immediately after the closed-form derivation that yields $A_\infty = 2$, a paragraph has been added stating that "the value $A_\infty = 2$ in (eq21) is therefore not an unconditional result but a conditional one … an unproven yet clearly bounded claim: unproven because non-overlap has not been established for all n; clearly bounded because, under the non-overlap assumption, the closed-form geometric series gives the explicit value 2 with no remaining freedom." The paragraph cross-references §6.3 for further discussion.
In §6.3, the corresponding paragraph has been expanded so that the same characterization is repeated and unpacked along two readings of "clearly bounded": first structurally (under non-overlap, the geometric series gives the explicit value 2 with no remaining freedom), and second measure-theoretically (without non-overlap, countable subadditivity of two-dimensional Lebesgue measure ensures that the bump-area series is in any case a finite upper bound on the limiting area, since the area of a union of measurable sets is at most the sum of their individual areas). The two readings agree under non-overlap and bracket the value otherwise. The value $A_\infty = 2$ is therefore stated as exact under non-overlap and as a finite upper bound on the limiting area without it; this stronger conditional statement is given in §3.4 (paragraph after eq21) and discussed in §6.3.
The closing sentence of §6.3 records that establishing or refuting non-overlap for the construction of §3.4 at arbitrary iteration depth is left as future work.
The same conditional framing has been propagated to §7 (Conclusion), where the value $A_\infty = 2$ is now described as "closed-form and numerically explicit, contingent on a stated geometric assumption that is verified visually for low iterations but not proved for arbitrary iteration depth."
Location in round-2 revised manuscript: §3.4 (paragraph immediately following eq21); §6.3 (second paragraph beginning "The status of the value $A_\infty = 2$…"); §7 (paragraph beginning "The Koch-style construction on a square presented in §3.4 illustrates…").

Reviewer comment 3 (verbatim)

"I think you are underselling your contribution. While I agree that no new theorem, condition, regime discovery……is made; the framework does function as a diagnostic tool to infer asymptotic perimeter and area behavior without re-deriving each example. Please also see the attached pdf chatgpt file for more defensible claims within the framework. I would like you to reframe your contribution precisely in terms of novelty in formulation and synthesis (and pedagogy). The chatgpt file also includ rewrite of the abstract and conclusion after such reframing. You can rewrite in your own words and terms."

Response

This comment has been adopted as the structural principle of the round-2 revision. The contribution has been reframed throughout the manuscript from "pedagogical" to *formulation and synthesis*, expressed in three concrete elements: (i) a *change of representation* in which two derived growth ratios $\alpha = N/r$ and $\beta = N/r^2$ become the primary control parameters; (ii) a *synthesis* that locates the four canonical examples within a single coordinate system rather than analyzing them in isolation; and (iii) a *construction-class refinement* that records a structural non-equivalence inside the same dimension class. The framework is presented as a *diagnostic tool* in the precise sense requested by the reviewer: given (N, r) and a specification of construction class, the asymptotic behavior of perimeter and area follows directly.

The reviewer's attached ChatGPT-file suggestions for the abstract and conclusion have been reviewed and adopted in substance. The author has rewritten both passages in the author's own register, retaining the structural moves (reframing as representation, introducing α and β as named ratios, presenting the additive/subtractive distinction as a structural non-equivalence inside the same dimension class, describing the framework as a diagnostic) and integrating them with the manuscript's existing scope statements and conditional-result language. The word "pedagogical" has been removed from the abstract, the introduction, and the conclusion; it does not appear anywhere in the round-2 manuscript.

The specific changes are as follows.

Abstract (rewritten in full). The abstract now opens by naming the contribution as "a unified parameter-space representation," introduces α and β as the principal control parameters, presents the regime classification as a *joint* asymptotic classification of perimeter and area, and characterizes the construction-class refinement as a "structural non-equivalence inside the same dimension class that is not visible from D alone." The closing sentence states that "the contribution of the paper lies in formulation and synthesis rather than in new mathematics: it consolidates several classical results into a single diagnostic representation in which, given (N, r) and a specification of construction class, the asymptotic behavior of perimeter and area can be inferred directly without re-deriving each example."

Introduction §1 (contribution paragraph rewritten and a new paragraph added). The aim-of-the-paper paragraph now describes the goal as "a unified parameter-space representation" rather than "a unified pedagogical framework." A new paragraph immediately following it states the contribution explicitly in three labeled elements (change of representation; synthesis; construction-class refinement) and concludes with the diagnostic-use framing.

§4.1 (renamed to "Scaling laws and derived growth ratios"; α and β introduced). The named ratios $\alpha = N/r$ and $\beta = N/r^2$ are introduced as the framework's primary control parameters in eqs. (eq22a)–(eq22b). The relation between α and similarity dimension D is recorded explicitly ($D = 1 + \log(\alpha)/\log(r)$), as is the observation that β is not a function of D — which is the technical reason the (α, β) coordinates carry information that D alone does not.

§4.2 (rewritten in α, β notation). Each of the asymptotic-behavior cases for perimeter and for area is now stated as an inequality on α or β with the equivalent (N, r) inequality in parentheses. A closing paragraph makes the structural point explicit: "a single inequality $\alpha > 1$ governs perimeter divergence, a single inequality $\beta < 1$ governs area boundedness, and the regime classification of §4.3 follows from the joint behavior of these two ratios."

§4.3 (last paragraph expanded). The contribution claim has been rewritten so that the regime classification is presented as a *joint* classification of two observables (perimeter behavior and area behavior) rather than as a corollary of the single-observable classical condition $1 < D < 2$. The boundaries $N = r$ and $N = r^2$ are explicitly equated with $\alpha = 1$ and $\beta = 1$, and the framework's structural point is stated directly: "The boundaries themselves are not new; their use here as the axes of a joint classification of perimeter and area is the framework's structural point."

§4.4 (rewritten and a "structural non-equivalence" paragraph added). The additive/subtractive distinction is now framed as a structural non-equivalence inside the same dimension class: "Two constructions can have identical (N, r), identical α, identical β, and identical similarity dimension D, yet exhibit qualitatively different asymptotic two-dimensional Lebesgue measure depending on which construction class they belong to." The paragraph notes that the (N, r, construction class) triple carries information that D alone does not, and identifies this distinction as one of the framework's principal organizing observations.

§4.5 (new subsection: "Diagnostic use of the framework"). A new subsection has been added that summarizes the content of §§4.1–4.4 as an explicit diagnostic procedure: given a deterministic self-similar planar construction in the class defined in §2.1, asymptotic perimeter and area are determined by α (perimeter), β (area), construction class (zero-versus-positive-finite Lebesgue measure within the intermediate regime), and the non-overlap assumption (closed-form additive area). The section ends by stating that "the framework's contribution is, in this sense, a consolidation of multiple individual derivations into a single diagnostic structure indexed by (N, r) and construction class."

§7 (Conclusion rewritten in full). The conclusion has been rewritten to mirror the abstract's reframing. The opening paragraph names the representation contribution and the diagnostic use of the framework. A second paragraph states the construction-class refinement as a structural non-equivalence inside the same dimension class. A third paragraph names the contribution explicitly as "one of formulation and synthesis rather than of new mathematics," with three labeled elements (change of representation; synthesis; construction-class refinement) and the diagnostic framing. A fourth paragraph records the conditional status of $A_\infty = 2$ (responding jointly to comment 2 above). The closing paragraph returns to the original motivating question and answers it in the diagnostic register.

The reviewer's instruction "you can rewrite in your own words and terms" has been followed: the substance of the suggested rewrite has been adopted, but the language is the author's, integrated with the manuscript's prior register, scope statements, and citation pattern. Phrases the reviewer's attached file flagged as off-limits — "new theorem," "new condition," "novel discovery of regime," and any implication that $r < N < r^2$ is itself new — do not appear in the round-2 manuscript.

Location in round-2 revised manuscript: Abstract (rewritten); §1 contribution paragraph and following new paragraph; §4.1 (renamed, with eqs. eq22a–eq22b added); §4.2 (rewritten in α, β); §4.3 (closing paragraph expanded); §4.4 (rewritten with structural-non-equivalence paragraph); §4.5 (new subsection); §7 Conclusion (rewritten).

Summary table — coverage of each round-2 comment

Comment	Section(s) of round-2 revised manuscript
Reviewer 1 (acceptance of physics-diffusion pushback)	No change required; §6.2 retained from round 1
Reviewer 2 (non-overlap assumption as unproven yet clearly bounded claim)	§3.4 (paragraph after eq21); §6.3 (expanded second paragraph); §7
Reviewer 3 (reframe contribution: representation, synthesis, diagnostic; abstract and conclusion rewrite)	Abstract; §1; §4.1 (renamed, α/β added); §4.2; §4.3; §4.4; §4.5 (new); §7

The author has reviewed all responses against the editor's notice that "if the the concerns/comments/questions of the reviewer are addressed inadequately, incompletely or insufficient address is paid to detail, your manuscript may be rejected." The author thanks the editor and reviewer again and remains available to address any remaining concerns.

Pedro Marotta 2026-05-04

Thank you for addressing my comments. As a last comment to show that the framework actually works beyond the canonical examples in section 3 that reproduce know results (post hoc interpretation), please show that, given a new construction, the model can infer correct behavior without re-derivation. To that effect, please include at least one example in each of the three regimes ($N \leq r$, $r < N < r^2$, $N \geq r^2$), and at least one pair of constructions with identical (N, r) but different construction class, to demonstrate that the framework produces distinct predictions under identical scaling parameters.

Response to Reviewer (Round 3) — Manuscript 3136520

Manuscript title: *A Scaling-Parameter Framework for Perimeter and Area in Self-Similar Planar Fractals*

Author: Pedro Marotta

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The author thanks the editor (Shireesh Apte) and the reviewer for the round-3 comment. The single comment received in this round is reproduced verbatim below, followed by a description of the response and the specific section(s) of the round-3 revised manuscript in which the response is implemented.

Reviewer comment 1 (verbatim)

"Thank you for addressing my comments. As a last comment to show that the framework actually works beyond the canonical examples in section 3 that reproduce know results (post hoc interpretation), please show that, given a new construction, the model can infer correct behavior without re-derivation. To that effect, please include at least one example in each of the three regimes ($N \le r$, $r < N < r^2$, $N \ge r^2$), and at least one pair of constructions with identical (N, r) but different construction class, to demonstrate that the framework produces distinct predictions under identical scaling parameters."

Response

This comment has been adopted in full. A new subsection §4.6 *"Predictive application to new constructions"* has been added, in which the framework is applied predictively — i.e. predictions are stated first, on the basis of the diagnostic of §4.5 alone, and then confirmed by direct calculation — to four further constructions outside the canonical four of Section 3. The four constructions cover the three regimes identified in §4.3 (one example per regime) and additionally include a pair with identical (N, r) but differing construction class, exactly as the reviewer requested.

The four constructions are as follows.

§4.6.1 — Subcritical regime ($N \le r$): a subtractive (N, r) = (2, 3) construction. The initial unit square is partitioned at each iteration into a 3 × 3 grid of nine sub-squares of side 1/3, and only two of the nine (the upper-left and lower-right corners) are retained; the rule is applied recursively. The growth ratios are $\alpha = 2/3 < 1$ and $\beta = 2/9 < 1$, with similarity dimension $D = \log(2)/\log(3) \approx 0.631 < 1$. The framework predicts $P_n \to 0$ and $A_n \to 0$ from §4.2 and the subtractive case of §4.4. Direct substitution into (eq22) and (eq24) gives $P_n = 4 \cdot (2/3)^n$ and $A_n = (2/9)^n$, confirming both predictions.

§4.6.2 — Intermediate-dimension regime ($r < N < r^2$): a new additive (N, r) = (6, 4) construction. Each boundary segment of length ℓ is divided into four equal parts of length ℓ/4; on the third part (counted along the segment's direction), an outward square of side ℓ/4 is constructed; the original third part is then removed. The segment is therefore replaced by six segments of length ℓ/4 (the first part, the second part, three sides of the outward square, the fourth part). The growth ratios are $\alpha = 3/2$ and $\beta = 3/8$, with similarity dimension $D = \log(6)/\log(4) \approx 1.292$. The framework predicts perimeter divergence and a positive finite asymptotic area under non-overlap. Direct calculation gives $P_n = 4 \cdot (3/2)^n \to \infty$ and $A_\infty = 1 + 2/5 = 7/5 = 1.4$ (under the non-overlap assumption recorded in §3.4 and §6.3, applied here in the same conditional sense to this construction).

§4.6.3 — Supercritical regime ($N \ge r^2$): an additive (N, r) = (10, 3) construction. Each boundary segment of length ℓ is replaced by ten segments of length ℓ/3 in some specified outward bump pattern, with the bumps assumed to satisfy the non-overlap convention so that the additive area model of (eq23) applies. The growth ratios are $\alpha = 10/3$ and $\beta = 10/9 > 1$, with similarity dimension $D = \log(10)/\log(3) \approx 2.10$. The framework predicts unbounded growth of the iteration-n perimeter P_n and unbounded growth of the additive area-counting series A_n (in which bump areas are counted with multiplicity at every iteration). Direct calculation gives $P_n = 4 \cdot (10/3)^n \to \infty$ and divergence of the additive series in (eq23), since the bump-area term at iteration k is proportional to $(10/9)^{k-1}$ and fails to converge. The §4.6.3 paragraph notes explicitly that the framework's prediction is the divergence of this additive area-counting series and not, in the supercritical regime, an unconditional statement about the actual two-dimensional Lebesgue measure of any planar realization (which depends on whether bumps overlap and is, by countable subadditivity, in any case bounded above by the additive series).

§4.6.4 — Identical (N, r) and dimension, differing construction class: a (5, 3) pair. The Koch-style construction on a square presented in §3.4 (additive, $A_\infty = 2$ under non-overlap) is paired with a (5, 3) subtractive construction in which the unit square is partitioned at each iteration into a 3 × 3 grid of nine sub-squares of side 1/3 and only five of the nine (the four corners and the centre) are retained; the rule is applied recursively. Both constructions share $\alpha = 5/3$, $\beta = 5/9$, and similarity dimension $D = \log(5)/\log(3) \approx 1.465$. The framework predicts identical perimeter behavior in the two cases ($P_n = P_0 \cdot (5/3)^n \to \infty$) but, by the construction-class refinement of §4.4, qualitatively different asymptotic two-dimensional Lebesgue measure: $A_\infty = 2$ (additive case, conditional on non-overlap) versus $A_\infty = 0$ (subtractive case, by direct substitution into (eq24): $A_n = (5/9)^n \to 0$). The pair therefore exhibits the structural non-equivalence of §4.4 directly: identical (N, r), identical α, identical β, and identical D yield qualitatively different asymptotic Lebesgue-measure outcomes solely because of differing construction class. The (N, r) pair, and equivalently D alone, are insufficient to predict the area outcome; the (N, r, construction class) triple is required.

Each of the four examples follows the predict-then-verify structure requested by the reviewer: the framework's diagnostic outputs (regime, perimeter behavior, area behavior, asymptotic Lebesgue measure) are stated first, on the basis of (N, r, construction class) alone, and a direct geometric-series calculation is then given that confirms each prediction. The four examples consist of three newly introduced constructions (the subtractive (2, 3) construction of §4.6.1, the additive (6, 4) construction of §4.6.2, and the additive (10, 3) construction of §4.6.3) together with one new paired comparison (§4.6.4), in which the previously analyzed Koch-style construction on a square of §3.4 is set against a newly introduced subtractive (5, 3) construction in order to isolate the construction-class refinement of §4.4 at fixed (N, r). The diagnostic is therefore exercised on constructions that lie outside the canonical four of Section 3 — either through introduction of a new rule or through a new pairing — and not as post-hoc re-derivation of behavior already established in Section 3.

The presence of §4.6 has been signalled in three additional places of the manuscript:

- the **Abstract** has been updated with a clause noting that "four further constructions are analyzed predictively to demonstrate that the framework's diagnostic outputs follow from (N, r, construction class) without re-derivation: one example in each of the three regimes, plus a pair with identical (N, r), and therefore identical similarity dimension, that nevertheless exhibit qualitatively different asymptotic area outcomes because of differing construction class";
- the **§1 roadmap paragraph** has been updated to note that Section 4 "applies the framework predictively (§4.6) to four further constructions outside the canonical four of Section 3 — one in each of the three regimes plus a pair with identical (N, r) and differing construction class";
- the **§7 Conclusion** has been extended with a paragraph summarizing the four §4.6 examples and stating that "the framework's predictions follow from (N, r, construction class) and are subsequently confirmed by direct calculation, so that the framework is exercised predictively and not only post hoc."

Location in round-3 revised manuscript: §4.6 (new subsection, with §§4.6.1–4.6.4 covering the subcritical, intermediate, supercritical regimes and the identical-(N, r) pair respectively); Abstract (clause added); §1 (roadmap paragraph updated); §7 (additional paragraph after the contribution summary).

Summary table — coverage of the round-3 comment

Comment	Section(s) of round-3 revised manuscript
Reviewer 1 (predictive examples: one per regime, plus an identical-(N, r) pair)	§4.6 (new), with §§4.6.1, 4.6.2, 4.6.3, 4.6.4; Abstract; §1 roadmap; §7

The author has reviewed the additions against the editor's notice that "if the the concerns/comments/questions of the reviewer are addressed inadequately, incompletely or insufficient address is paid to detail, your manuscript may be rejected." The author thanks the editor and reviewer again and remains available to address any remaining concerns.

Pedro Marotta 2026-05-05

Thank you for addressing my comments. Accepted.