Peer-Review

Zhang, Leo. 2026. “Structural Equivalence and Novelty in Constrained Tiling Sequences: A Computational Classification Framework.” *Journal of High School Science* 10 (2): 101–22. https://doi.org/10.64336/001c.160815

1.Not being in OEIS does NOT mean a sequence is new because it might exist under a different indexing, a different interpretation or as a transformed form. What you have stumbled upon (rediscovered) - but have failed to recognize -is that you are dealing with a classical lattice walk enumeration problem with a return-to-origin constraint. This sits in the family of random walks, central binomial structures and diagonal generating functions. Hence, your examples are not new; they merely represent special cases of constrained random walks / lattice path theory. The paper is not about tilings—it is about counting constrained lattice walks via rational generating functions. Mapping to lattice walks usually means the result is not new, but already implicitly known. Your results may map even more generally or non-locally to multivariate generating functions, diagonal of rational functions, Markov chains or stochastic processes, species or symbolic combinatorics, multinomial contingency table counting. Every sequence in the paper is a projection of the same underlying combinatorial structure, viewed through different choices of step encoding, weights and endpoint constraints. Your primary contribution is illustrative - not theoretical.
You have overstated novelty. Therefore, instead of claiming “….lead to unique sequences”, which is probably false; you should claim "lead to diverse and structurally rich sequences, many of which can be interpreted within the framework of constrained lattice walks and generating function diagonals.”
On checking OEIS novelty, please perform the following. Normalize the sequence by shift index, divide by constants, remove zeros or rescale length. Indentify the generating function, search by structure, not numbers and compare asymptotics.
2.The term ‘small restriction’ is not formally defined and may be misleading, as - in reality - even a single linear constraint significantly alters the combinatorial and asymptotic structure of the problem.” by lambda^n tends to lamba^n/squareroot(n). This is a major structural change. You are therefore not looking at ‘small’ restrictions by any means.
PIVOT ONE
3. Your paper does not contribute to the general corpus of knowledge in this field UNLESS, you can turn your results on their head and propose “A sequence arising from a constrained tiling model is not structurally novel if it admits multiple independent realizations that reduce to the same lattice walk model under the framework above.” In other words, if your (OEIS) sequence has has multiple independent combinatorial realizations, it is typically not fundamentally new, but a member of a known structural class.
You can hence claim a method (reverse engineering) of classification that determines when sequences are genuinely new vs when they are reinterpretations.
4.Here’s a novelty and experience section that you can include in your manuscript (after pivoting from your original thesis and claim):
Your new title can be something like “We formalize a practical criterion for distinguishing genuinely new sequences from reinterpretations within a known combinatorial framework.”
A natural question arising from our constructions is whether the sequences obtained from constrained tilings are genuinely new, or whether they represent previously studied combinatorial objects in disguise. In this section, we formalize a criterion to distinguish between novel sequences and new interpretations of known structures.
6.1 Reverse Engineering Framework
Given a sequence ((a_n)) defined via a tiling model with constraints on tile counts, we may associate to each tile type (t_i):

a length ($w_i \in \mathbb{Z}_{>0}$),
a constraint weight ($c_i \in \mathbb{Z}$),
and interpret any valid tiling as a sequence satisfying:

$$\sum_i w_i k_i = n, \qquad \sum_i c_i k_i = r,$$

where (k_i) is the number of tiles of type (t_i), and (r) is a fixed constant determined by the constraint.
This induces a correspondence with lattice walks of length (n) whose steps are drawn from a set

$$S = \{(w_i, c_i)\},$$

with the condition that the walk terminates at height (r). Thus, each constrained tiling model can be reinterpreted as a lattice walk with endpoint constraints.

6.2 Equivalence of Combinatorial Models

We say that two combinatorial models are equivalent if they induce the same counting sequence ((a_n)) and correspond to the same class of lattice walks up to:
relabeling of steps,
linear rescaling of weights,
or reparameterization of indices.
Under this definition, many superficially different tiling models may in fact be equivalent representations of the same underlying structure.

6.3 Criterion for Structural Novelty

We propose the following criterion:
A sequence arising from a constrained tiling model is not structurally novel if it admits multiple independent realizations that reduce to the same lattice walk model under the framework above.
In particular, if a sequence can be reverse engineered into:
a lattice walk with step set ($\{-1,0,+1\}$) and endpoint constraint, or
a multinomial counting problem with linear constraints,
then it belongs to a well-studied class of combinatorial objects (e.g., constrained random walks or diagonal generating functions), and should be regarded as a new interpretation rather than a new structure.

6.4 Application to Our Sequences

Applying this criterion to the sequences introduced earlier:
The sequence (e_n), defined via equal numbers of squares and trominos, corresponds to lattice walks with steps ($-1, 0, +1$) returning to the origin. This admits multiple equivalent formulations and matches a known sequence in the OEIS.
The sequence (i_n), defined via equal numbers of red and orange dominos, yields the same underlying walk model up to rescaling of step lengths, and is likewise not structurally distinct.
The sequence (j_n), where the number of red dominos exceeds the number of orange dominos by one, corresponds to walks ending at height ($+1$), and is therefore a shifted instance of the same class.
The sequence (g_n), while not immediately identified in the OEIS, can be interpreted as a constrained composition or weighted lattice walk, suggesting that it belongs to the same general family.

6.5 Implications

This analysis shows that many sequences arising from “simple restrictions” are not isolated phenomena, but rather instances of a broader combinatorial framework. Consequently, the primary contribution of such constructions lies in:
providing new combinatorial interpretations,
illustrating connections between tilings and lattice walks,

and motivating further exploration of constrained enumeration problems.
6.6 Conclusion of Novelty Analysis
We conclude that the sequences derived in this paper are best understood as elements of a unified class of constrained lattice walk enumerations. While some sequences may not be explicitly listed in existing databases, their structural properties indicate that they are not fundamentally new objects, but rather new representations of known combinatorial principles.
6.You will need to add :a short worked example of reverse engineering step-by-step. You can start with your sequences, go back and 'discover' they they can be derived from multiple new representations of known combinatorial principles.
PIVOT 2
7. Instead of saying "Here are some sequences", you can say "We show that constrained tiling sequences fall into universality classes determined by the number of independent constraints, and that many seemingly distinct constructions are equivalent under lattice-walk representations". Specifically, you would prove that the type of constraint determines the asymptotic growth rate. You would hence actually calculate the growth rate with one..two…three..four… linear constraints on counts with fixed length tiles and derive a universal asymptotic principle. This will explain why all your sequences behave similarly. It will go beyond examples → gives a general law using accessible tools such as Stirling's approximation and binomial coefficients.
I would prefer you incorporate both pivots.

Reviewer 1:

- **Comment: Not being in OEIS does NOT mean a sequence is new because it might exist under a different indexing, a different interpretation or as a transformed form. What you have stumbled upon (rediscovered) - but have failed to recognize -is that you are dealing with a classical lattice walk enumeration problem with a return-to-origin constraint. This sits in the family of random walks, central binomial structures and diagonal generating functions. Hence, your examples are not new; they merely represent special cases of constrained random walks / lattice path theory. The paper is not about tilings—it is about counting constrained lattice walks via rational generating functions. Mapping to lattice walks usually means the result is not new, but already implicitly known. Your results may map even more generally or non-locally to multivariate generating functions, diagonal of rational functions, Markov chains or stochastic processes, species or symbolic combinatorics, multinomial contingency table counting. Every sequence in the paper is a projection of the same underlying combinatorial structure, viewed through different choices of step encoding, weights and endpoint constraints. Your primary contribution is illustrative - not theoretical.**

You have overstated novelty. Therefore, instead of claiming "….lead to unique sequences", which is probably false; you should claim "lead to diverse and structurally rich sequences, many of which can be interpreted within the framework of constrained lattice walks and generating function diagonals."

On checking OEIS novelty, please perform the following. Normalize the sequence by shift index, divide by constants, remove zeros or rescale length. Indentify the generating function, search by structure, not numbers and compare asymptotics.

- Addressed by clarifying diction throughout the paper and reframing the thesis
- **Comment: The term 'small restriction' is not formally defined and may be misleading, as - in reality - even a single linear constraint significantly alters the combinatorial and asymptotic structure of the problem." by lambda^n tends to lamba^n/squareroot(n). This is a major structural change. You are therefore not looking at 'small' restrictions by any means.**
- Addressed by removing instances of this throughout the paper
- **Comment: Entire block about Pivot 1 (too bulky to list verbatim)**
- Introduced Section 6 to address this comment, basically took the given structure and expanded upon it

- **Comment: Instead of saying “Here are some sequences”, you can say “We show that constrained tiling sequences fall into universality classes determined by the number of independent constraints, and that many seemingly distinct constructions are equivalent under lattice-walk representations”. Specifically, you would prove that the type of constraint determines the asymptotic growth rate. You would hence actually calculate the growth rate with one..two…three..four… linear constraints on counts with fixed length tiles and derive a universal asymptotic principle. This will explain why all your sequences behave similarly. It will go beyond examples → gives a general law using accessible tools such as Stirling’s approximation and binomial coefficients.**
- Introduced Section 5 to address this comment

Reviewer 2:
- **Comment: Please verify that all links to the references point to the correct source.**
- Double checked that
- **Comment: The authors must disclose and acknowledge any assistance received in the preparation of this manuscript, including but not limited to editorial, technical, analytical, or writing support. All such contributions must be clearly stated in the Acknowledgments section.**
- Added missing Acknowledgements section before the References section
- **Comment: Include enough recent references along with foundational ones. Ensure references directly support your claims. Avoid “padding.” Use credible sources (peer-reviewed journals, reputable books, official reports).**
- Added more comprehensive references

Thank you for addressing my comments. The manuscript is much improved. However, you have addressed my comments more in conceptual level rather than at the rigor level. Please address the comments below.

1.Clarify and appropriately scope the “universal asymptotic principle”

The current presentation overstates the level of rigor. The result is presented as a general principle, but it is supported only by heuristic reasoning and selected examples.

Required actions:

Reframe this result as a proposition or conjectural pattern, not a proven general law.

Clearly state the scope and assumptions under which it is expected to hold.

Include 2–3 fully worked representative cases that demonstrate the claimed behavior.

Add a short empirical validation section, showing numerical evidence that supports the proposed scaling behavior.

Explicitly acknowledge the heuristic nature and limitations of the argument.

2.Make the novelty criterion operational

The current “criterion for structural novelty” is conceptually sound but not practically usable.

Required actions:

Convert the criterion into a step-by-step procedure that a reader can follow.

Present this as an explicit algorithm or checklist, including:

normalization of sequences

identification of constraint structure

mapping to an equivalent combinatorial model

comparison with known classes

Include one complete worked example demonstrating the procedure from start to finish.

Clarify what constitutes a positive vs negative determination of novelty.

3.Strengthen and formalize the equivalence framework

The current definition of equivalence remains informal and lacks clear boundaries.

Required actions:

Provide a precise definition of when two models are considered equivalent.
Clearly specify:
which transformations are allowed
what properties must be preserved
Add a short subsection on limitations of the equivalence notion, including:
ambiguous cases
potential edge conditions
Ensure that all equivalence claims made later in the paper are consistent with this definition.
4.Improve OEIS comparison and normalization methodology
The current approach to identifying known sequences is insufficiently rigorous and not reproducible.
Required actions:
Add a dedicated subsection describing a systematic procedure for sequence comparison, including:
normalization steps (indexing, scaling, removal of trivial terms)
structural comparison (recurrences, generating behavior, asymptotics)
Avoid relying solely on matching initial terms.
Provide at least one example where normalization is necessary to correctly identify equivalence.
Emphasize structure-based identification, not just numerical matching.
5.Better situate the work within existing combinatorics literature
The manuscript currently lacks sufficient engagement with established theory and appears somewhat isolated.
Required actions:
Add a short section explicitly connecting the work to:
lattice path enumeration
generating function methods
known combinatorial counting frameworks
For each connection, briefly explain how the present work relates to existing results.
Strengthen citations by referencing standard sources, not only databases or informal summaries.
Reframe the contribution as a reinterpretation and synthesis within known frameworks, rather than an isolated development.
6.Extension to modular or nonlinear constraints
The current work focuses exclusively on exact linear balance constraints. A natural extension would be to consider modular or periodic variants of these constraints, where balance is enforced only up to a fixed modulus rather than exactly.
Suggested direction:
Replace exact equality conditions with modular balance conditions (e.g., requiring counts to agree modulo a fixed integer).
Analyze how this modification affects:
the structure of the counting sequences
the form of the generating functions
the asymptotic behavior
What this would add:
Tests the robustness of the current framework under relaxed constraints
May reveal periodic or oscillatory effects in sequence behavior
Provides a bridge between strict combinatorial constraints and more flexible counting models
This extension can likely be handled using the same structural mapping approach already developed in the paper.
7.Incorporation of local constraints (forbidden patterns / defects)
The current models impose only global constraints on tile counts. A complementary extension would be to introduce local restrictions, such as forbidding certain adjacent tile configurations.
Suggested direction:

Define tiling models where specific local patterns are disallowed
Analyze the resulting sequences using finite-state or transition-based methods
Compare the growth behavior and classification of these sequences with the unrestricted case
What this would add:
Introduces a richer class of models combining global and local constraints
Allows investigation of how sensitive the proposed framework is to structural perturbations
Provides a more realistic and broadly applicable combinatorial setting
This direction remains computationally and conceptually accessible while significantly strengthening the scope of the paper.

Clarify and appropriately scope the “universal asymptotic principle”
- Moved universal asymptotic principle (UAP) to section 7, included the acknowledgement of limitations, scope, and assumptions near the end of the first subsection of section 7
- Reformatted existing sections into worked cases (7.1, 7.2)
- Added empirical validation (7.3)

Make the novelty criterion operational
- Algorithm introduced in section 5.4
- Completely worked example in section 5.5 and section 5.6
- Clarified novelty definition in beginning of section of 5.4

Strengthen and formalize the equivalence framework
- Sections 5.2 and 5.3 address all comments

Improve OEIS comparison and normalization methodology
- A newly introduced section 5.5 addresses this comment

Better situate the work within existing combinatorics literature
- Added a section 6 to address this comment

Extension to modular or nonlinear constraints
- Addressed in newly added section 8.1

Incorporation of local constraints (forbidden patterns / defects)
- Addressed in newly added section 8.2

Again; you have answered my concerns superficially. I will put it to you bluntly:
You are trying to design an algorithm. If I feed in tiling sequences (or operations that lead to specific tiling sequences), the algorithm should tell me one of two things: 1. Yes, the sequence is novel and does not exist/cannot be derived from/reduced to ones in the database or 2. the sequence is not novel and exists in / can be reduced to one that already exists in the database.
Where is this algorithm in the manuscript? and where are its results?
Right now, what you call an “algorithm” is not really a strict, push-button algorithm. It’s more like a set of guidelines and/or a step-by-step thought process, rather than something that always works, always gives a clear yes/no answer and is falsifiable.
Please present an algorithm that works in a manner as in**the attached chatgpt file.** Feed it 30 tiling sequences and evaluate it based on accuracy, false positives, false negatives, true positives, true negatives. Your algorithm must be available in the public domain if and when your paper is accepted.

Algorithm concern:
- Addressed in sections 5.3 to 5.7

Thank you. The paper is improved from its earlier version. However, you still do not adequately distinguish between what part of the algorithm is heuristic and what may be construed as ‘universal’; nor do you explicity list the steps to derive it. You also do not present metrics for evaluation of your algorithm. To that effect, please find attached a chatgpt blurb on how to rewrite section 5. Please use this as a template and rewrite section 5 accordingly.

You also overrreach with “we propose a heuristic asymptotic proposition dictating that the macroscopic growth rate of a tiling sequence is strictly governed by the number of independent linear constraints, which universally forces a polynomial damping factor of 1/n^{k/2}.”. Please temper your claim to "We propose a unifying heuristic interpretation: in constrained tiling models, each independent linear constraint induces a Gaussian suppression factor, leading to asymptotic growth of the form (write your damping factor). This observation is consistent with known results from lattice-walk asymptotics and analytic combinatorics, but provides a simple operational rule for predicting universality classes directly from combinatorial constraints.

Algorithm concern:
- Rewrote entirety of section 5
- Added sequences to Appendix A

Heuristic asymptotic proposition:
- Reworded at the end of the initial portion of Section 7 (right before the beginning of Section 7.1)

Thank you for addressing my comments.

1.Please deposit your code and related auillaries to GitHub and provide the link in the manuscript under a section titled “Data availability”. The GitHub contents must include code that is ready to run, normalization procedure, recurrence detection method and model fitting details.

2.To remove circularity, please include this paragraph in an appropriate section - probably methods. "To avoid circularity in evaluating the classifier, we explicitly decouple the assignment of ground truth labels from the internal logic of the algorithm. The benchmark dataset is partitioned into two categories: (i) externally validated sequences, whose classification as “known/reducible” is established independently through prior literature and entries in the OEIS, and (ii) synthetically constructed sequences, whose generative rules are explicitly defined to include nonlinear, nonlocal, or arithmetic constraints that fall outside the reduction classes targeted by the classifier (e.g., not representable by binomial sums, D-finite recurrences, or standard lattice-walk models). For the synthetic set, the ground truth label “candidate novel” is assigned a priori based solely on the construction mechanism, not on the classifier’s output. The classifier is then applied in a strictly blind manner to all sequences, and its predictions are compared against these independently assigned labels. This separation ensures that performance metrics reflect the classifier’s ability to generalize beyond its defining assumptions, rather than merely reproducing them. "

3.Tone down unsupportable claims such as “conclusively demonstrate”…. etc. replace with softer verbiage such as “suggest”, “indicate”, “provide preliminary evidence”…. Please check the entire manuscript.

Github:
- Added Section 10
- All code in repository linked in Section 10

Remove circularity:
- Added paragraph in Section 5.5

Diction:
- Cleaned up diction in manuscript (sections 2, 6, 7), also removed outdated instances of reverse engineering framework diction

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Thank you for addressing my comments. Before I accept your manuscript, please upload a word or libreoffice writer document formatted to the Journal’s guidelines. Please make sure that all the equations transcribe correctly.

Doc formatting:
- Submitted a word .docx

- All equations made in Microsoft Word equation maker, on my preview end they do not display, but they do load when I open the file in Microsoft Word

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Thank you for addressing my comments. Accepted.