



A reproducible Burridge–Knopoff spring–block toy model with explicit numerical diagnostics for stick–slip dynamics

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Abstract

Earthquake faults spend most of their cycle locked while tectonic loading increases elastic stress, followed by rapid slip that releases stored strain. Spring-block models provide a transparent bridge between this mechanics and observable event statistics, but student-level implementations can suffer from numerical drift unless integration and event parsing are explicit. Here, we implement the standard Burridge–Knopoff spring-block framework as a reproducible, classroom-ready toy model with nondimensionalization, Euler–Cromer integration, and fixed event-detection thresholds. A 50-block chain reproduces the canonical stick–slip signature, and small random initial offsets preserve the same burst–quiet dynamics after a short transient, demonstrating robustness beyond symmetric conditions. In the single-slider limit, the friction gap controls recurrence timing; across multiple realizations, the mean inter-event interval scales approximately linearly with ΔF with quantified uncertainty. A minimal two-block system shows that elastic coupling promotes synchronization, captured by an event-coincidence measure. Within this framework, we test whether slip size correlates with subsequent waiting time, quantify biases from incomplete detection and measurement location, estimate a decorrelation horizon from divergence of nearly identical simulations, and demonstrate parameter degeneracy where distinct parameter sets yield identical mean recurrence but different slip sizes. Finally, we introduce a perturbation–response extension in which a localized disturbance during a stick phase is analyzed using the same event definitions to quantify cascade size, participation fraction, and amplification, linking the toy model to earthquake triggering and stress transfer. Appendices provide diagnostics and sensitivity checks.

Keywords

Burridge–Knopoff model, Stick-slip dynamics, Earthquake modeling, Spring-block system, Seismicity simulation, Frictional sliding, Recurrence time statistics, Elastic coupling, Computational physics, Earthquake triggering

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1. Introduction

1.1 Background

Earthquakes arise from sudden slip on faults that have been loaded slowly by tectonic plate motion. Although the Earth's crust is structurally complex, the underlying failure mechanism is conceptually clear: elastic strain accumulates during long periods of sticking, friction resists motion, and once the stored elastic force exceeds the fault's strength the system fails and releases energy rapidly. A major scientific challenge is connecting this simple physical picture to the diverse timing patterns and event clustering observed in seismicity, while keeping models transparent enough to test assumptions and interpret results. Spring-block models provide a powerful and longstanding bridge between mechanics and seismic statistics. In the Burridge-Knopoff representation (1,2), blocks approximate fault segments, springs represent elastic stiffness within and between segments, and a driving plate imposes slow loading; despite their idealization, such systems can generate realistic stick-slip cycles, intermittent bursts, and broad event-size behavior. A common statistical target for these toy models is frequency-magnitude scaling, often summarized by the Gutenberg-Richter relation (3). For student-level computational work, an additional challenge is numerical stability: slip events involve rapid velocity changes that can cause simple explicit schemes to drift or fail if implemented carelessly. To address this, we use the Euler-Cromer update, a stable modification of forward Euler that is widely recommended for spring-mass systems (4). Spring-block stick-slip has also been used as a computational teaching example in its own right (5), and recent

studies increasingly connect slider-block dynamics to data-driven and machine-learning approaches. Finally, simplified physics-based simulators, such as ShakeAlert® (<https://shakealert.org>), can serve as controlled testbeds for ideas relevant to earthquake monitoring and early warning systems (8). A related seismological question is how small stress perturbations from nearby earthquakes, fluid-pressure changes, or tidal forcing either die out locally or cascade into further failure. In that sense, spring-block models are useful not only for spontaneous recurrence statistics but also as controlled environments for thinking about stress transfer, earthquake triggering, and susceptibility to cascading rupture (14-18).

1.2 Summary

This study implements the standard Burridge-Knopoff spring-block model and focusses on reproducibility and pedagogy: one recommended parameter set, explicit numerical and threshold sensitivity checks, and a compact set of results that can be regenerated figure-by-figure with minimal tuning. Targeted simulations are then used to address three questions. First, can a minimally specified spring-block chain reproduce the canonical stick-slip cycle and its robustness to small symmetry-breaking perturbations? Second, how does the mean inter-event interval $\langle T \rangle$ scale with the friction gap ΔF in a single-slider reduction? Third, in a two-block system, how does elastic coupling k_c promote synchronization, as seen directly in displacement traces (Figure 8) and quantified by an event-coincidence measure (Figure 9)? By answering these questions with reproducible code and explicit diagnostics, the study provides

a compact platform for connecting classical fault-dynamics ideas to modern seismic analysis and warning research.

Importantly, we do not attempt to invent a new earthquake model; instead, we build on the well-known Burridge-Knopoff framework and ask what additional insight a carefully designed, student-accessible implementation can provide. The paper's contributions are therefore practical rather than "new physics." Using one recommended parameter set (Table 1), Figures 1-4 summarize a 50-block chain: Figure 1 shows the staircase-like displacement of the middle block (block 25), Figure 2 shows the corresponding velocity bursts, and Figures 3 and 4 repeat the displacement/velocity views under small random initial offsets to demonstrate robustness beyond perfect symmetry. Figures 5-7 isolate a single slider to quantify how the mean inter-event interval scales with the friction gap ΔF (Figure 7 reports Monte Carlo mean recurrence with a linear fit). Finally, Figures 8 and 9 use a two-block system to show how coupling controls synchronization, visualized directly in the displacement traces (Figure 8) and summarized via an event-coincidence metric (Figure 9; Appendix A2 contrasts this with cross-correlation lag). This organization is intended to make the model transparent, reproducible, and suitable for classroom exploration while keeping the limits of a simplified toy model explicit. In addition, to include some elements of originality while remaining within the same transparent toy framework, we include four extension analyses that go beyond reproducing canonical stick-slip: (i) an event-size versus subsequent-waiting-time test (Figure 10), (ii) an observational-bias

analysis showing how size-based detection thresholds reshape inferred recurrence statistics (Figure 11), (iii) a predictability-horizon estimate from divergence of nearly identical simulations (Figure 12), and (iv) two complementary bias/identifiability demonstrations: location-dependent statistics along the chain (Figure 13) and a simple degeneracy example showing that distinct parameter combinations can match the same mean recurrence while producing different slip sizes (Figure 14). We also make explicit a perturbation-response extension in which a localized kick applied during a stick phase is translated into cascade-size and amplification observables, so the same model can speak not only to recurrence under steady loading but also to susceptibility to triggering.

2. Methods

2.1 Theory

The governing equations follow the standard Burridge-Knopoff spring-block model. We nondimensionalize the equations so that all reported quantities and plotted axes are explicitly scaled. We define the characteristic time $t_0 = \sqrt{m/k_p}$, displacement $x_0 = F_s/k_p$, velocity $v_0 = x_0/t_0$, and force scale F_s . Using $t^* = t/t_0$, $x^* = x/x_0$, $v^* = v/v_0$, and $F^* = F/F_s$, the model is written and simulated in nondimensional form. For readability we drop the asterisks, and label axes using the explicit ratios (e.g., t/t_0 , x_{25}/x_0 , $v_{25}/(x_0/t_0)$). A chain of N blocks with displacements x_i is driven by a plate at $u(t) = Vt$, coupled elastically to neighbors by k_c , and governed by a static-kinetic friction switch with thresholds F_s and F_k . In the

single-slider limit, the friction gap $\Delta F = F_s - F_k$ sets the drive distance needed to reach the next failure: after a slip relaxes the resisting branch toward F_k , the driver must rebuild $\sim \Delta F$ of elastic force to reach F_s . Because the elastic loading rate during sticking scales as $k_p V$, a back-of-the-envelope estimate is $\langle T \rangle \approx \Delta F / (k_p V)$ (in units of t_0). In dimensional terms, $v_{\text{phys}} = v(x_0/t_0)$ and $T_{\text{phys}} = T t_0$. For example, choosing $x_0 = 1 \text{ cm}$ and $t_0 = 0.5 \text{ s}$ (illustrative), a plotted dimensionless velocity $v = 0.4$ corresponds to $0.4 x_0/t_0 \approx 0.008 \text{ m/s}$, and a recurrence interval $\langle T \rangle \approx 40$ corresponds to about 20 s. These conversions are intended only as an interpretation guide; mapping the toy model to a specific fault would require a separate calibration of $(m, k_p, F_s, \dots)$ with deviations expected from finite slip duration and multi-block interactions. This scaling motivates the linear fit reported in Figure 7 and provides a physically interpretable link between a friction parameter and recurrence timing (3). The Euler-Cromer update is used for time integration because it is a stable modification of forward Euler for spring-mass dynamics and suppresses spurious numerical energy drift in long runs (4).

2.2 Simulation

This section specifies the numerical update, event-detection rules, and robustness checks used throughout, with all parameter values and tested ranges recorded in Table 1. The objective is not to maximize realism by tuning, but to provide a schedule-ready workflow that other students can reproduce figure-by-figure with

minimal ambiguity.

2.2.1 Baseline toy-model parameters

Unless stated otherwise, we use the single recommended parameter set in Table 1. We set $N = 50$ for the chain figures (Figures 1-4), $N = 1$ for the friction-gap sweep (Figures 5-7), and $N = 2$ for the synchronization demonstration (Figures 8 and 9). We use $\Delta F = 0.20$ (equivalently, $F_k = 0.40$ with $F_s = 0.60$) for the two-block synchronization figures (Figures 8 and 9) to obtain a denser sequence of slip events within a short display window; unless stated otherwise, all other parameters match Table 1. The plate speed V sets the loading rate. The friction gap $\Delta F = F_s - F_k$ is the control parameter for recurrence-time scaling in the single-slider study, while the coupling stiffness k_c is the control parameter for synchronization in the two-block study. Boundary conditions and the initial-condition variants are listed explicitly in Table 1.

The recommended baseline configuration consists of a 50-block chain with each block of mass $m = 1$, plate-spring stiffness $k_p = 1.0$, and nearest-neighbor coupling stiffness $k_c = 0.3$. Friction is represented by a two-level law with static threshold $F_s = 0.6$ and kinetic level $F_k = 0.2$, so that the friction gap is $\Delta F = F_s - F_k = 0.4$. The driver plate moves at constant speed $V = 0.02$, which, together with the friction gap, sets the typical recurrence time. The time step is $\Delta t = 0.01$, small enough to resolve the short slip phases while keeping the runs computationally inexpensive on a laptop.

Table 1. Recommended toy-model parameters for the spring–block fault system. All quantities are nondimensional as described in Section 2.1. Baseline values are used unless a range is specified; sensitivity sweeps are listed in the rightmost column.

Symbol	Meaning	Baseline	Units	Range tested / notes
N	Number of blocks	50	-	Main runs use $N = 50$; Appendix A4 compares $N = 25, 50$, and 100.
m	Block mass	1.0	-	Set by nondimensionalization (kept fixed).
k_p	Plate-block spring stiffness	1.0	-	Set to 1.0 as a reference stiffness for nondimensional ratios.
k_c	Neighbor coupling stiffness	0.30	-	Sensitivity / synchronization sweep: $k_c/k_p \in \{0, 0.1, 0.2, 0.3, 0.6, 0.9, 1.2\}$ ($k_c = 0$ gives independent sliders).
V	Plate speed (loading rate)	0.020	-	Appendix A3 sweeps $V \in \{0.010, 0.015, 0.020, 0.030, 0.040\}$ (single-slider).
F_s	Static friction threshold	0.60	-	Held fixed in all runs shown.
F_k	Kinetic friction level	0.20	-	Varied to set $\Delta F = F_s - F_k$; see Fig. 7 and Fig. 14.
ΔF	Friction gap ($F_s - F_k$)	0.40	-	Fig. 7 uses $\Delta F \in \{0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50\}$.
Δt	Time step	0.010	-	Convergence check: $\Delta t \in \{0.040, 0.020, 0.010, 0.005\}$.
V_{th}	Velocity threshold for event start/stop	2×10^{-3}	-	For event catalogs: $V_{th} = 2 \times 10^{-3}$. For illustrative velocity plots and the single-slider recurrence scaling, $V_{th} = 5 \times 10^{-3}$.
ε	Participation threshold used for defining chain-wide events	0.01	-	Participation threshold used for defining chain-wide events (where applicable).
t_q	Quiet time used to merge closely spaced pulses	1.0	time	Quiet time used to merge segments: activity separated by $\leq t_q$ is merged into one event.
Boundary	Boundary conditions	Open / free-end	-	First/last blocks couple to only one neighbor; interior blocks couple to two neighbors.

2.2.2 Numerical update (Euler-Cromer)

We integrate the equations of motion using the Euler-Cromer scheme (velocity update followed by position update), which improves numerical stability for oscillatory spring-mass dynamics

relative to naive forward Euler (4). With time step Δt , at each step $n \rightarrow n+1$, we first compute the elastic forces $F_i^{spr}(x^n, t_n)$, then update each block as

$$v_i^{n+1} = v_i^n + \frac{\Delta t}{m} (F_i^{spr} - F_i^{fric}), \quad x_i^{n+1} = x_i^n + v_i^{n+1} \Delta t.$$

The stick test is applied first to enforce genuinely flat "stuck" intervals: if $|F_i^{spr}| \leq F_s$ and $|v_i^n| \leq v_{stick}$, we set $v_i^{n+1} = 0$ and $x_i^{n+1} = x_i^n$. Otherwise, the block slips with kinetic friction $F_i^{fric} = F_k \operatorname{sgn}(v_i^n)$. Appendix A provides a controlled oscillator test (Figure A1) to illustrate

the stability advantage of this semi-implicit update. We verify time-step convergence by repeating representative runs at smaller Δt values listed in Table 1; once Δt is sufficiently small, both the qualitative stick-slip pattern and the reported summary statistics change negligibly.

2.2.3 Event detection, observables, and uncertainty estimates

Slip events are extracted from velocity time series using a threshold v_{th} (Table 1). For the chain, an event begins when $\max_i |v_i(t)|$ first exceeds v_{th} and ends when all blocks return below v_{th} ; for the single slider, the same definition is applied to $|v(t)|$. To avoid splitting a single physical slip into multiple numerical micro-events, threshold crossings separated by less than a quiet time t_q are merged. For multi-block measures, a displacement threshold ϵ is used to classify whether a block meaningfully participates in an event. From the resulting event catalog we compute: (i) displacement and velocity time series; (ii) inter-event (waiting) times T ; (iii) event sizes $S = \sum_i |\Delta x_i|$ and a classroom-friendly magnitude proxy $m = \log_{10} S$; and (iv) synchronization metrics based on event coincidence for the two-block study (with Appendix A2 using cross-correlation lag as a cautionary comparison). For Figure 7, each ΔF value is evaluated with

multiple independent realizations after discarding an initial transient, and we report the Monte Carlo mean $\pm$ standard error for $\langle T \rangle$; the least-squares fit parameters and per-point sample sizes are reported in the Figure 7 caption and Results paragraph. As a roadmap of the results, Figures 1-4 summarize the $N=50$ chain by tracking the middle block (block 25): baseline behavior (Figures 1 and 2) and robustness under small random initial offsets (Figures 3 and 4). Figures 5-7 isolate a single slider ($N=1$) to quantify how the mean inter-event interval $\langle T \rangle$ scales with the friction gap $\Delta F = F_s - F_k$. Figures 8 and 9 use a two-block system ($N=2$) to show how coupling stiffness k_c controls synchronization, visualized directly in displacement traces (Figure 8) and summarized with an event-coincidence metric (Figure 9).

2.2.4 Modeling

A fault segment is represented with a Burridge-Knopoff spring-block toy model: a one-dimensional chain of N identical blocks with displacements $x_i(t)$ sliding on a rough surface while a driver plate advances steadily. The driver plate position is $u(t) = Vt$. Each block is attached to the plate by a spring of stiffness k_p and elastically coupled to its nearest neighbors by springs of stiffness k_c . For an interior block i ($2 \leq i \leq N-1$), the net elastic force is given by,

$$F_i^{spr} = k_p(u - x_i) + k_c(x_{i+1} - x_i) + k_c(x_{i-1} - x_i).$$

We use open (free-end) boundary conditions: the first and last blocks couple to only one neighbor, hence $F_1^{spr} = k_p(u - x_1) + k_c(x_2 - x_1)$ and $F_N^{spr} = k_p(u - x_N) + k_c(x_{N-1} - x_N)$.

Friction is modeled by a static-kinetic switch. While a block is stuck, friction cancels the elastic pull up to the static threshold F_s . Once the threshold is exceeded, the block slips and

experiences kinetic friction of magnitude $F_k < F_s$ opposing motion; during slip the equation of motion is $m \ddot{x}_i = F_i^{spr} - F_k \operatorname{sgn}(\dot{x}_i)$.

All parameters are reported in dimensionless form. We adopt one recommended baseline parameter set (Table 1) so that a reader can reproduce every figure without retuning, and we vary only one control parameter at a time in each results subsection. In the full-chain demonstrations ($N=50$), we visualize the dynamics by tracking the middle block (block 25). To test robustness beyond perfect symmetry, we also run a heterogeneous-initial condition variant by adding small random offsets to $x_i(0)$ (Table 1), which primarily shifts the timing of the earliest events without changing the qualitative stick-slip pattern.

2.3 Perturbation-response extension

While the results in Section 3 focus on spontaneous events under steady loading, the same event-detection framework can be used to study controlled perturbations without introducing new observables. Specifically, during a loading (stick) phase we can apply a small localized displacement perturbation $\delta x/x_0 \ll 1$ to a single block and then parse the resulting response with the same velocity threshold v_{th} , quiet-time merge window t_q , and participation threshold ε already used throughout Table 1. Because the catalog definitions are unchanged, perturbation-triggered cascades can be compared directly with the baseline spontaneous-event statistics.

For such an experiment, a natural cascade size is $S_{\text{cascade}} = \sum_i |\Delta x_i|$, where Δx_i is the displacement increment accumulated by block i during the

triggered response. A corresponding participation fraction is $P = (1/N) \sum_i H(|\Delta x_i| - \varepsilon)$, where H is the Heaviside step function, and an effective amplification can be summarized by $\chi = E(PN)$, i.e., the expected number of blocks activated by a local perturbation. These quantities reuse the same slip-size and participation ideas already introduced for spontaneous events, but redirect them toward propagation rather than recurrence.

This extension also clarifies several results already reported here. The parameter degeneracy in Figure 14 suggests that systems with the same mean recurrence may still be distinguishable by χ or by the distribution of S_{cascade} . Likewise, the location dependence in Figure 13 implies that perturbations introduced near chain ends versus the interior need not trigger the same response. Finally, the predictability experiment in Figure 12 measures divergence under infinitesimal perturbations, whereas the present protocol would examine finite-amplitude perturbations at the event level. We therefore include this perturbation-response framework as an explicit extension of the existing model rather than as a separate new model.

3. Results

All figures use the nondimensional variables defined in Section 2.2.1. Axes explicitly state the scaling (e.g., t/t_0 , x/x_0 , $v/(x_0/t_0)$, δ_{rms}/x_0 , and $\langle T \rangle/t_0$). Unless otherwise noted, parameters such as $\Delta F = F_s - F_k$ and k_c/k_p are dimensionless.

3.1 Baseline chain behavior

Using the baseline parameter set in Table 1 and

constant initial conditions $x_i(0)=0$ and stress accumulates, and each rapid step $v_i(0)=0$, the middle block (block 25) in the $N=50$ chain exhibits a clear stick-slip cycle. The displacement trace $x_{25}(t)$ (Figure 1) advances in a staircase-like pattern: long plateaus correspond to sticking while elastic loading reaches the static friction threshold F_s . This baseline run provides a clean, reproducible reference trajectory for later robustness and scaling tests.

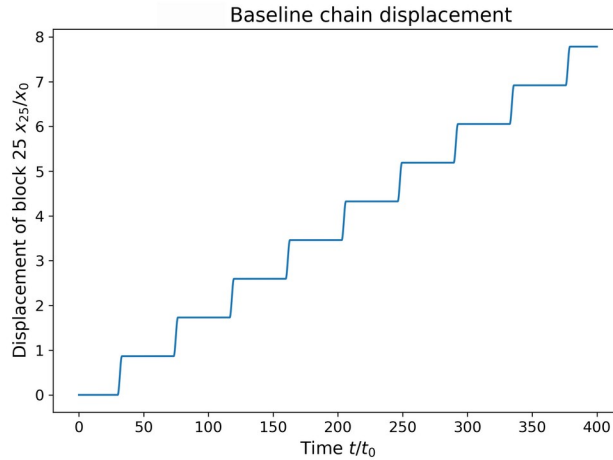


Figure 1. Baseline displacement time series for a 50 -block chain (block 25 shown). Axes: time t/t_0 and displacement x_{25}/x_0 .

3.2 Velocity view of baseline stick-slip

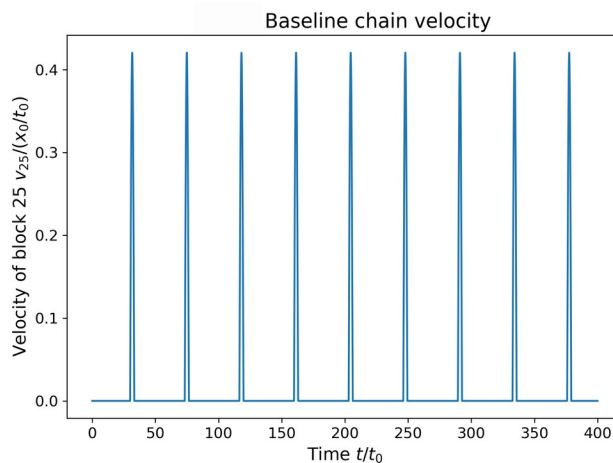


Figure 2. Baseline velocity time series for the same 50 -block chain (block 25 shown). Axes: time t/t_0 and velocity $v_{25}/(x_0/t_0)$.

Figure 2 shows the corresponding velocity $v_{25}(t)$ for the same baseline run. During sticking, $v_{25}(t)$ remains extremely close to zero, confirming that the block is effectively locked while stress rebuilds. Slip events appear as narrow, high-amplitude bursts, which makes event timing and recurrence measurements well-posed: using a fixed detection threshold $v_{th} = 5 \times 10^{-3}$ (Table 1) reliably separates fast slip from slow loading without subjective tuning

3.3 Robustness of stick–slip to irregular initial stress

To test whether the baseline pattern is an artifact of a perfectly symmetric start, we repeat the chain simulation using the same parameters as Figure 1 (Table 1) but with small random initial offsets in x . The displacement x (Figure 3) remains staircase-like, with slip steps and

sticking plateaus still clearly separated. The main qualitative behavior therefore persists under heterogeneous prestress, which is closer to real fault conditions where stored stress is not spatially uniform. The small overshoot visible at intermediate times in Figure 3 occurs immediately after a large slip step and reflects short-lived elastic "ringing" in the mass-spring system: inertia and coupling can carry the block slightly past its eventual post-event position before the system re-locks and relaxes during the subsequent stick phase. This overshoot is therefore a physically consistent transient within the toy dynamics rather than a numerical instability; in that limited sense it is analogous to dynamic overshoot discussed for natural ruptures, although no direct quantitative mapping is claimed here (19). It does not change the event-by-event recurrence measurements reported later in the paper.

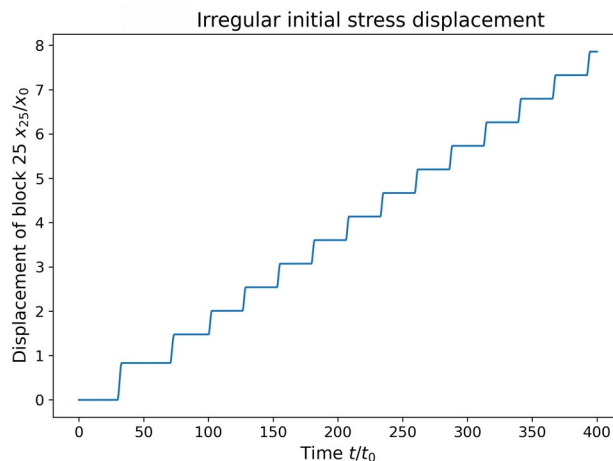


Figure 3. Displacement time series for a 50-block chain with small symmetry-breaking initial offsets (block 25 shown). Axes: t/t_0 and x_{25}/x_0 .

3.4 Velocity under irregular initial stress

Figure 4 plots the velocity $v_{25}(t)$ for the same irregular-start run as Figure 3. In this particular

realization, the apparent decrease in peak velocity with time is primarily a transient associated with relaxation from the initially

imposed heterogeneous prestress: the earliest events can be slightly larger as the chain adjusts, after which the event amplitudes become statistically steady. For the "irregular initial stress" case (Figures 3 and 4), we use independent random initial displacement offsets $x_i(0) \sim N(0, \sigma^2)$ with $\sigma = 10^{-3}$ (dimensionless) and set $v_i(0) = 0$ (fixed RNG seed for reproducibility). Figures 3 and 4 show a single representative run (not an average over realizations); Monte Carlo averaging is used

only where explicitly stated (e.g., Figure 7). Slip events remain concentrated into short, high-amplitude spikes separated by long near-zero intervals, indicating that the stick-slip cycle remains well-defined even when the system does not begin from a symmetric state. For consistent event parsing across runs, we use $v_{th} = 5 \times 10^{-3}$ together with a quiet-time merge window $t_q = 1.0$ (Table 1), so a brief sub-threshold dip does not split a single physical slip episode into multiple events.

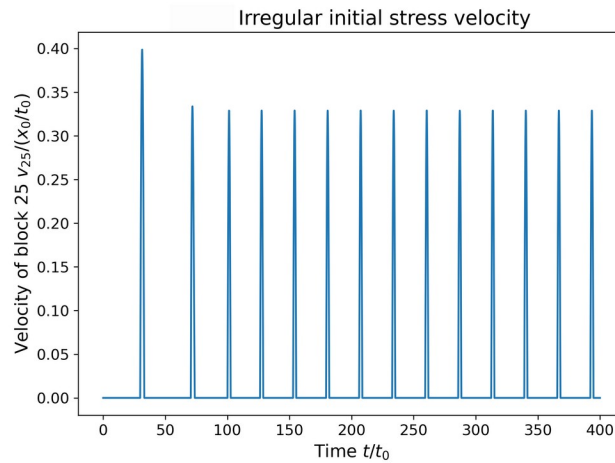


Figure 4. Velocity time series for a 50 -block chain with small symmetry-breaking initial offsets (block 25 shown). Axes: t/t_0 and $v_{25}/(x_0/t_0)$.

3.5 Canonical stick-slip cycle in a single degree of freedom

Before interpreting the multi-block dynamics, it is useful to isolate the core mechanism in the simplest setting. Figures 1 and 2 track the motion of one block embedded in an interacting $N=50$ chain, hence each slip is influenced by stress transfer from neighbors and by occasional multi-block participation; this naturally produces less regular event sizes and timing than an isolated oscillator. In contrast, Figures 5 and 6 show the $N=1$ single-slider reduction (with no

neighbor coupling), which highlights the canonical stick-slip mechanism in its cleanest, nearly periodic form. The qualitative differences between the chain and single-slider plots therefore reflect the expected transition from coupled to single-degree-of-freedom dynamics rather than any inconsistency in definitions. In the single-slider system ($N=1$), the plate spring loads the block at constant rate V until the net driving force reaches the static threshold F_s ; the block then fails, friction transitions toward the kinetic level F_k , and rapid slip occurs. The

resulting displacement $x(t)$ (Figure 5) cleanly providing an interpretable baseline that separates slow loading from fast release, underlies the more complex behavior observed in coupled chains.

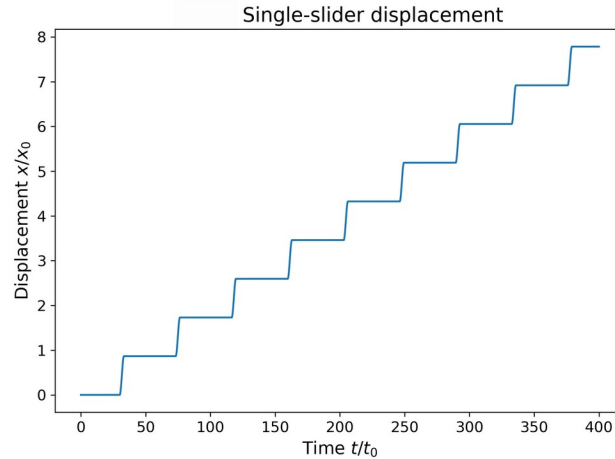


Figure 5. Single-slider displacement time series ($N = 1$). Axes: t/t_0 and x/x_0 .

3.6 Kinematics of single-slider failure

Figure 6 presents the corresponding velocity $v(t)$ for the same single-block run. Slip events appear as sharp velocity bursts, framed by long intervals of rest while stress rebuilds. This velocity view motivates the operational event

definition used throughout the paper: event start and stop times can be identified reproducibly by a fixed threshold v_{th} (Table 1), enabling fair comparisons of recurrence time and slip statistics across parameter sweeps.

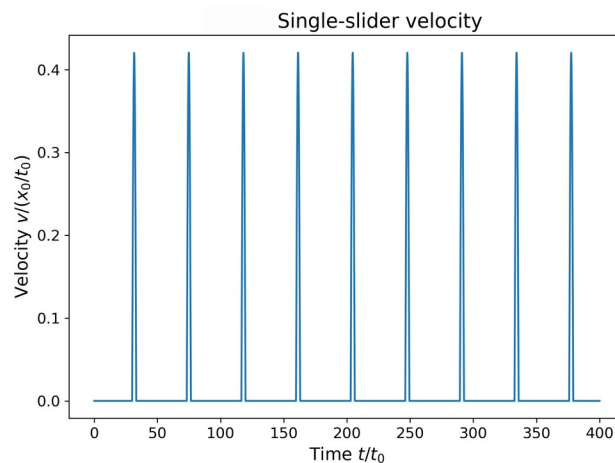


Figure 6. Single-slider velocity time series ($N = 1$). Axes: t/t_0 and $v/(x_0/t_0)$.

3.7 Recurrence-time scaling from Monte Carlo sweeps

To quantify how the friction gap $\Delta F = F_s - F_k$ controls recurrence time, we study the single-slider limit ($N=1$) driven at the baseline loading rate $V=0.020$ with $k_p=1$ and $k_c=0$. For each tested $\Delta F \in \{0.15, 0.20, \dots, 0.50\}$, we run 25 independent realizations with small random initial offsets, discard the first five events as transient, and compute the mean start-to-start inter-event interval $\langle T \rangle / t_0$. Figure 7 shows the

ensemble mean with ± 1 standard error (SE) error bars. Each point is the mean over 25 realizations; standard errors are smaller than the marker size. Over this range, $\langle T \rangle / t_0$ increases $\sim$ linearly with ΔF ; an OLS fit gives $\langle T \rangle / t_0 \approx 99.52 \Delta F + 3.44$ ($R^2 > 0.99$). Because $\Delta F \rightarrow 0$ removes the static-kinetic gap, the intercept is not physically meaningful in this idealized setting; we use the fit only to summarize the near-linear scaling.

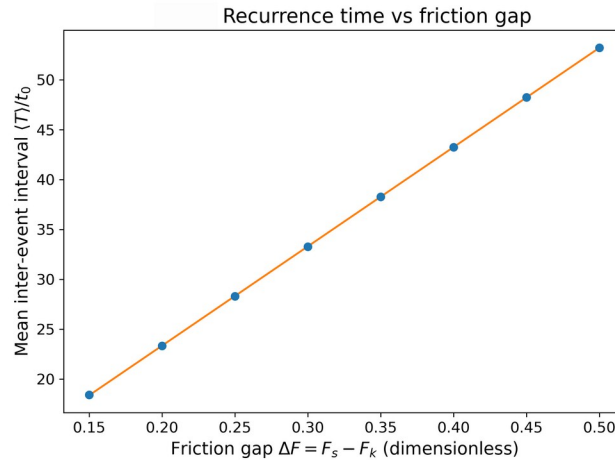


Figure 7. Mean recurrence interval $\langle T \rangle / t_0$ versus friction gap $\Delta F = F_s - F_k$ (dimensionless) for the single-slider model. Each point is the mean over 25 realizations; error bars show ± 1 SE and are smaller than the marker size. Linear fit: $\langle T \rangle / t_0 \approx 99.52 \Delta F + 3.44$ ($R^2 > 0.99$).

3.8 Synchronization driven by elastic coupling

Figure 8 shows the displacements of two blocks ($N=2$) coupled by an elastic spring of stiffness k_c . When k_c is small, stress transfer is weak and

the blocks can slip at noticeably different times. As k_c increases, the motion of one block increasingly loads its neighbor, which promotes coordinated failure and pulls the motions into a quasi-coherent unit.

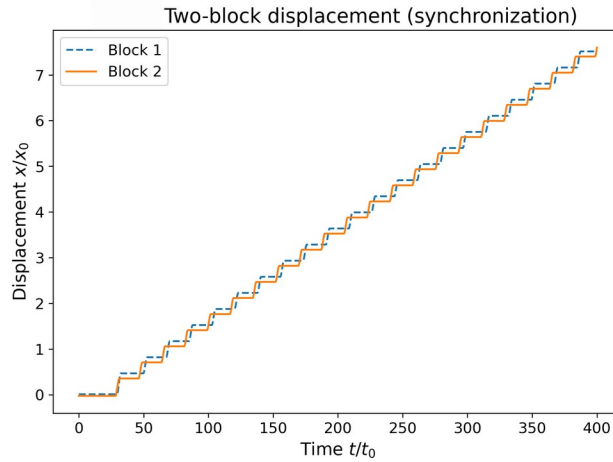


Figure 8. Two-block displacement time series ($N = 2$) illustrating synchronization. Axes: t / t_0 and x / x_0 .

3.9 Quantifying synchronization with event coincidence

To quantify synchronization in a way that matches the physical idea of "slipping together," we measure event coincidence rather than relying only on a cross-correlation lag.

Appendix A2 (Figure A2) provides a compact illustration of why a lag-only metric can remain near $\tau \approx 0$ even when slip events are not strongly coincident, motivating the event-based coincidence measure used here.

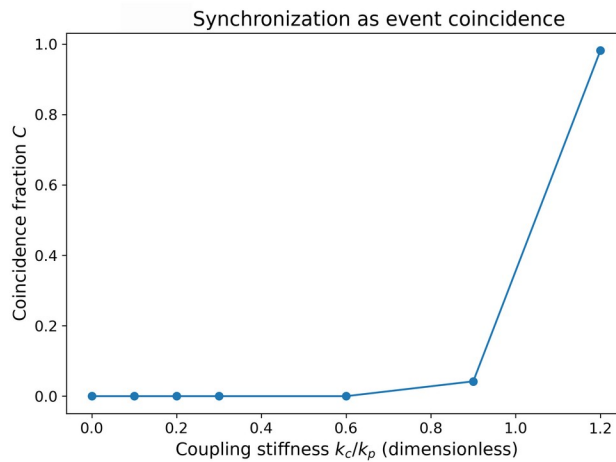


Figure 9. Synchronization quantified by an event coincidence fraction C (start-time matching window $w = 1.0$ in units of t / t_0) as a function of coupling stiffness ratio k_c / k_p .

For each block, we define slip-event start times using the same fixed velocity threshold v_{th} and quiet-time merging window t_q (Table 1). We then count a pair of events as "coincident" if the

two start times fall within a coincidence window $w = 1.0$ (dimensionless time), consistent with the event-merging rule. The coincidence fraction C (number of paired events normalized by the total number of events in both catalogs) increases with coupling stiffness k_c , directly capturing the idea that coupling raises the probability of joint slip. This also explains why a lag metric can be misleading at small k_c : long stick phases and common plate loading can keep the overall displacement time series broadly aligned (yielding a near-zero best-lag) even when actual slip events rarely coincide. We use a two-block system here because it is the minimal setting that isolates coupling as the only mechanism promoting coordination; it provides a transparent demonstration before introducing the additional complexity of an extended chain, consistent with prior Burridge-

Knopoff studies of coupling and coordinated slip (5,6).

3.10 Slip size versus subsequent waiting time

Because we analyze a local displacement time series, we define a simple slip-size proxy for block 25: for each detected event, $S = \Delta x_{25} / x_0$, the displacement increment accumulated during that event. We then measure the subsequent start-to-start waiting time $\Delta t_{\text{start}} / t_0$ to the next event on the same block (event detection uses $V_{\text{th}} = 2 \times 10^{-3}$ and quiet-time merging $t_q = 1.0$). Pooling 8 independent realizations with small random initial offsets ($n = 280$ event pairs), Figure 10 shows that larger slips tend to be followed by longer waits; an OLS fit yields $R^2 > 0.95$ (Pearson $r = 0.977$). We treat this as an empirical relationship within the toy model rather than a universal scaling law.

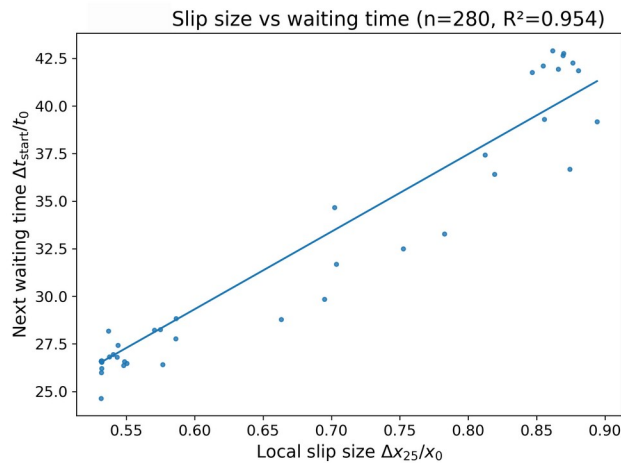


Figure 10. Local slip-size proxy Δx versus subsequent start-to-start waiting time T for a 50-block chain (pooled event pairs). OLS fit yields the relation reported in the main text (Pearson r shown there).

3.11 Observation bias under incomplete detection

In practice, not all slips may be detected; raising the minimum detectable size S_{min} biases the

inferred recurrence upward because small events are skipped and their waiting times are effectively merged. To quantify this, we recompute the mean waiting time using only

events with $S \geq S_{\min}$, with $S_{\min}$ chosen as conditional mean $\langle T \rangle(S_{\min})/t_0$; error bars are quantiles (90%, 95%, and 98%) of the slip-size bootstrap standard errors, illustrating the distribution in Figure 10. Figure 11 plots the systematic bias introduced by incomplete detection.

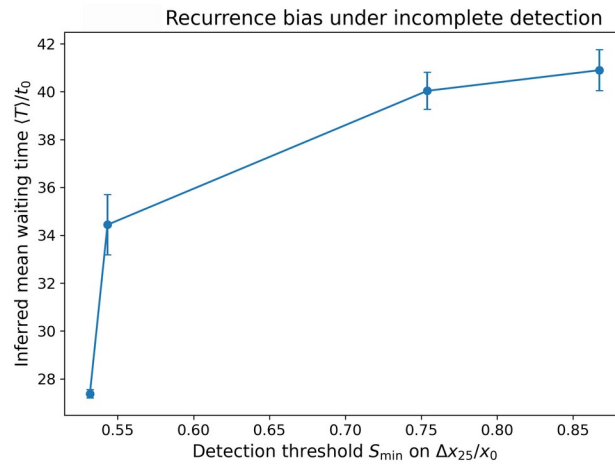


Figure 11. Detection-threshold bias: inferred mean waiting time $\langle T \rangle/t_0$ increases when only events above a size threshold $S_{\min}$ are retained. Error bars are bootstrap standard errors.

3.12 Predictability horizon from trajectory divergence

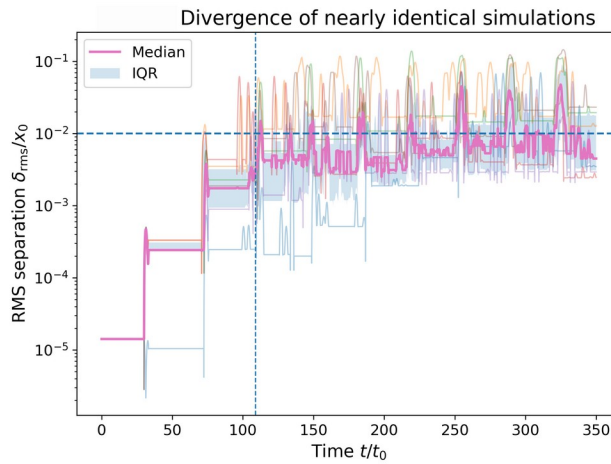


Figure 12. Predictability experiment: RMS separation δ_{rms}/x_0 between two simulations that differ only by a tiny initial perturbation ($\Delta x = 10^{-4}x_0$ on one block). Curves show multiple realizations; median and IQR are highlighted. Dashed line marks $\delta_{\text{rms}}/x_0 = 10^{-2}$ and median crossing time (~ 109.0 in units of t/t_0).

To probe predictability limits, we run paired simulations with identical parameters and identical initial conditions except for a tiny displacement perturbation applied to one block. We track the RMS separation, defined as the square root of the mean squared block-by-block displacement difference, normalized by x_0 . Figure 12 shows this separation for an ensemble of 10 realizations (different random initial offsets). The separation remains small for a finite time and then grows, crossing the chosen decorrelation threshold at a median of ~ 109.0 (in units of t_0 ; interquartile range shaded). We interpret this crossing time as an operational decorrelation horizon for state prediction under this perturbation size; it is not a Lyapunov-

exponent estimate, and it does not imply deterministic predictability beyond that horizon.

3.13 Location-dependent observational bias along the chain

In a finite chain, the event catalog inferred from a single observation point can differ from the chain-average behavior. To illustrate, we compute local recurrence times from blocks 1, 25, and 50 across multiple realizations of the $N=50$ chain with small random initial offsets. Figure 13 shows that the inferred mean waiting time $\langle T \rangle / t_0$ depends on observation location (end blocks versus the middle), even with identical global parameters. This motivates caution when inferring system-wide parameters from local records.

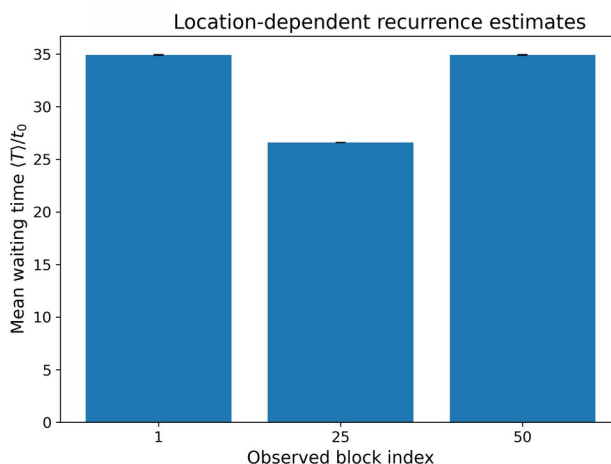


Figure 13. Location-dependent recurrence estimates in a finite chain. Bars show mean waiting time $\langle T \rangle / t_0$ inferred from local catalogs at blocks 1, 25, and 50; error bars are ± 1 SE across realizations.

3.14 Degeneracy in summary statistics (non-uniqueness of parameter inference)

In Figure 14, we show that matching one summary statistic does not uniquely identify underlying parameters, even in the single-slider limit. Different parameter combinations can

reproduce the same mean recurrence time. This has direct earthquake relevance because recurrence statistics alone can hide important differences in hazard. In particular, [1] Two faults may exhibit similar average recurrence intervals, [2] One fault may release stress

through many small events, while another does so through fewer, larger ruptures [3]. If only recurrence statistics are considered, these faults may appear similar despite having very different hazard implications.

Figure 14 therefore demonstrates that distinct parameter choices can yield the same average recurrence time while producing systematically different slip magnitudes, illustrating a fundamental degeneracy in inference based on

recurrence statistics alone. In the single-slider limit, Case A ($\Delta F = 0.30$, $V = 0.015$) and Case B ($\Delta F = 0.40$, $V = 0.020$) both yield essentially the same mean recurrence $\langle T \rangle / t_0$ (≈ 43.24) but different mean slip per event (≈ 0.648 versus 0.865 in units of x/x_0). This motivates the perturbation-response framework introduced in Section 2.3: two systems that look similar under steady loading may still differ in cascade susceptibility.

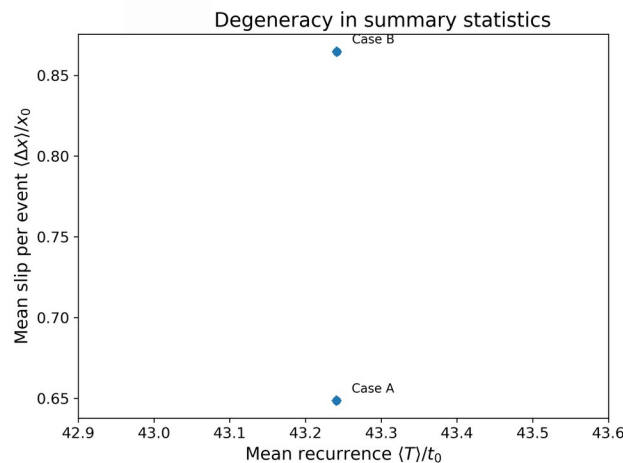


Figure 14. Degeneracy in summary statistics (non-uniqueness of parameter inference) in the single-slider limit. Two parameter sets yield nearly the same $\langle T \rangle / t_0$ but different mean slip per event $\langle \Delta x \rangle / x_0$.

4. Discussion

This paper does not propose a new earthquake-forecasting model. Instead, it distills the standard Burridge-Knopoff spring-block framework into a reproducible, classroom-ready toy model and uses it to illustrate practical challenges that arise when interpreting event catalogs from an idealized stick-slip system. The revised analysis emphasizes: (i) recurrence-time scaling with the static-kinetic gap ΔF supported by expanded Monte Carlo sampling with uncertainty estimates (Figure 7); (ii)

synchronization diagnostics based on event coincidence rather than lag alone (Figures 8 and 9 and Figure A2); (iii) the dependence of inferred statistics on detection thresholds and observation location (Figures 11 and 13); (iv) an operational predictability (decorrelation) horizon quantified via ensemble divergence under tiny perturbations (Figure 12); and (v) an explicit perturbation-response extension that links localized disturbances to cascade susceptibility and earthquake triggering. Appendices A3 and A4 extend the exploration

to loading-rate and system-size sensitivity checks, and Appendix A5 provides distribution-level summaries of slip sizes and waiting times to complement the time-series views.

4.1 Perturbation-response framework and links to real-world seismology

The perturbation-response framework provides a natural bridge between the simplified spring-block dynamics studied here and real earthquake systems, where the key question is not merely when failure initiates but whether it propagates. In natural faults, small stress perturbations arising from nearby earthquakes, fluid injection, or tidal forcing can either dissipate locally or promote cascading rupture depending on elastic coupling and stress heterogeneity. That behavior is central to modern interpretations of earthquake triggering and Coulomb stress transfer (14,15,18). Within this context, the amplification metric χ can be interpreted as a toy-model analogue of the branching ratio used in statistical seismology (16,17): low- χ systems respond mostly with isolated events, whereas high- χ systems are more cascade-prone and can approach system-spanning failure.

Framed this way, the extension yields several quantitative predictions that are consistent with the mechanisms already explored under steady loading. The mean amplification χ should increase with coupling stiffness k_c/k_p , larger systems should allow larger maximum cascades and a sharper crossover between localized and global response, and stronger barriers to failure (larger ΔF) should suppress propagation.

Conversely, higher loading rates V should make it more likely that a perturbation encounters blocks already close to threshold. The cascade-size distribution is therefore expected to broaden from localized, rapidly decaying behavior in weakly coupled systems toward heavy-tailed behavior in strongly coupled chains. These are predictions of the extension framework, not new measured results claimed for the present manuscript.

4.2 Limitations and perspective

Several limitations should be kept explicit when interpreting these ideas in the context of natural faults. The friction law used here is a simplified static-kinetic switch rather than a rate-and-state constitutive law, so the perturbation-scaling arguments should be understood as qualitative trends rather than calibrated forecasting relations (20,21). The model is also one-dimensional and mostly homogeneous, whereas real fault systems contain geometric complexity, heterogeneity, fluids, and multiscale interactions (18). Future work should therefore test whether the predicted perturbation-response trends remain robust in heterogeneous or higher-dimensional BK systems and, eventually, compare model-derived cascade statistics with empirical measures of earthquake triggering such as aftershock productivity or inferred branching ratios from seismic catalogs (15-17). That next step would help determine whether susceptibility to local perturbations can distinguish fault systems that share similar recurrence statistics but differ in their capacity for cascading rupture.

Table 2. Links between model observables, laboratory behavior, and representative modern sources.

Prediction from the model	What to expect in labs	Representative modern sources
Stick-slip with fast velocity spikes	Sudden stress drops, rapid slip, distinct AE bursts	Heslot 1994 (9); Rubino 2017 (10)
Waiting time $\uparrow$ with friction gap / healing	Recurrence interval grows with frictional healing or larger $F_s - F_k$	Im 2017 (11)
GR-style size distribution	Power-law frequency magnitude of AE or slip sizes	Korkolis 2021 (3,12); Goebel 2017 (13)
Stronger coupling $\rightarrow$ more synchronous motion	More coherent slip between coupled sliders / analog BK chains	Erickson 2011 (5); Perinelli 2023 (6)

Table 2 summarizes common behaviors and measurable observables in spring-block simulations (stick-slip with fast velocity spikes, waiting-time control by frictional weakening, size-statistics concepts such as Gutenberg-Richter-style scaling, and synchronization by coupling) and lists representative studies that document each behavior. In this revised version, the same logic is also extended in the surrounding discussion to perturbation-response metrics that connect the toy model to earthquake triggering in natural faults.

5. Conclusion

A reproducible Burridge-Knopoff spring-block toy model designed to be implemented, tested, and extended with minimal ambiguity is presented. Using one recommended baseline parameter set (Table 1), we demonstrate: (i) baseline stick-slip signatures in a 50-block chain and their robustness to small random initial offsets (Figures 1-4); (ii) explicit nondimensionalization and event parsing so that all figures and derived statistics are consistently

interpreted within one scaled framework (Table 1; Figures 5 and 6; Appendix A); (iii) a recurrence-time scaling relation in the single-slider limit, reporting mean recurrence as a function of the friction gap ΔF with uncertainty estimates (Figure 7); and (iv) a minimal two-block synchronization study showing how k_c/k_p controls the probability of joint slip, quantified by an event-coincidence metric (Figures 8 and 9; Appendix A2). We also include compact extension analyses quantifying size-waiting-time coupling (Figure 10), observational bias from incomplete detection (Figure 11) and measurement location (Figure 13), an ensemble-based decorrelation horizon from trajectory divergence (Figure 12), degeneracy in mapping summary statistics back to parameters (Figure 14), and an explicit perturbation-response framework that defines cascade size, participation fraction, and mean amplification for localized kicks applied during the stick phase (Section 2.3; Section 4). Appendices A3-A5 provide loading-rate, system-size, and distribution-level sensitivity checks.

Data availability

The Python scripts used to generate the simulations, figures, and tables described in this manuscript are available at the GitHub repository at <https://github.com/ms3027/burridge-knopoff-model>

6. References

1. Burridge, R., Knopoff, L. (1967). Model and theoretical seismicity. *Bulletin of the Seismological Society of America*, 57, 341–371. <https://doi.org/10.1785/BSSA0570020341>
2. Carlson, J. M., Langer, J. S. (1989). Mechanical model of an earthquake fault. *Physical Review A*, 40, 6470–6484. <https://doi.org/10.1103/PhysRevA.40.6470>
3. Gutenberg, B., Richter, C. F. (1954). *Seismicity of the Earth and associated phenomena* (2nd ed.). Princeton University Press.
4. Cromer, A. (1981). Stable solutions using the Euler approximation. *American Journal of Physics*, 49, 455–459. <https://doi.org/10.1119/1.12550>
5. Erickson, B., Birnir, B., Lavallée, D. (2011). Periodicity, chaos and localization in a Burridge–Knopoff model of an earthquake with variable friction. *Geophysical Journal International*, 187, 1787–1804. <https://doi.org/10.1111/j.1365-246X.2011.05261.x>
6. Perinelli, A., Iuppa, R., Ricci, L. (2023). A scalable electronic analog of the Burridge–Knopoff model of earthquake faults. *Chaos*, 33, 093103. <https://doi.org/10.1063/5.0154822>
7. Borate, P., Rivière, J., Marty, S., Marone, C., Kifer, D., Shokouhi, P. (2024). Physics-informed neural network can retrieve rate and state friction parameters from acoustic monitoring of laboratory stick-slip experiments. *Scientific Reports*, 14, 24624. <https://doi.org/10.1038/s41598-024-24624-0>
8. Given, D. D., Cochran, E. S., Heaton, T. H., Hauksson, E., Allen, R. M., Hellweg, M., et al. (2018). Revised technical implementation plan for the ShakeAlert system—An earthquake early warning system for the West Coast of the United States (USGS Open-File Report 2018–1155). <https://doi.org/10.3133/ofr20181155>
9. Heslot, F., Baumberger, T., Perrin, B., Caroli, B., Caroli, C. (1994). Creep, stick-slip, and dry-friction dynamics: Experiments and a heuristic model. *Physical Review E*, 49, 4973–4988. <https://doi.org/10.1103/PhysRevE.49.4973>

10. Rubino, V., Rosakis, A. J., Lapusta, N. (2017). Understanding dynamic friction through spontaneously evolving laboratory earthquakes. *Nature Communications*, 8, 15991. <https://doi.org/10.1038/ncomms15991>
11. Im, K., Elsworth, D., Marone, C., Leeman, J. (2017). The impact of frictional healing on stick-slip recurrence interval and stress drop: Implications for earthquake scaling. *Journal of Geophysical Research: Solid Earth*, 122. <https://doi.org/10.1002/2017JB014211>
12. Korkolis, M. P., Qiu, Q., Ben-Zion, Y., Brace, W. F. (2021). Earthquake-like events in laboratory experiments: Observations, statistical analyses, and comparison with seismicity. *Pure and Applied Geophysics*, 178, 3389–3415. <https://doi.org/10.1007/s00024-021-02825-7>
13. Goebel, T. H. W., Kwiatak, G., Dresen, G. (2017). Seismic moment and b value variations during seismic sequences induced by hydraulic fracturing. *Geophysical Research Letters*, 44, 11674–11682. <https://doi.org/10.1002/2017GL075878>
14. Stein, R. S. (1999). The role of stress transfer in earthquake occurrence. *Nature*, 402, 605–609. <https://doi.org/10.1038/45144>
15. King, G. C. P., Stein, R. S., Lin, J. (1994). Static stress changes and the triggering of earthquakes. *Bulletin of the Seismological Society of America*, 84, 935–953. <https://doi.org/10.1785/BSSA0840030935>
16. Ogata, Y. (1988). Statistical models for earthquake occurrences and residual analysis for point processes. *Journal of the American Statistical Association*, 83, 9–27. <https://doi.org/10.1080/01621459.1988.10478560>
17. Helmstetter, A., Sornette, D. (2002). Subcritical and supercritical regimes in epidemic models of earthquake aftershocks. *Journal of Geophysical Research: Solid Earth*, 107, 2237. <https://doi.org/10.1029/2001JB001580>
18. Scholz, C. H. (2002). *The mechanics of earthquakes and faulting* (2nd ed.). Cambridge University Press.
19. Ide, S., Baltay, A., Beroza, G. C. (2011). Shallow dynamic overshoot and energetic deep rupture in the 2011 Tohoku-Oki earthquake. *Science*, 332, 1426–1429. <https://doi.org/10.1126/science.1207020>

20. Dieterich, J. H. (1979). Modeling of rock friction: 1. Experimental results and constitutive equations. *Journal of Geophysical Research: Solid Earth*, 84, 2161–2168.

<https://doi.org/10.1029/JB084iB05p02161>

21. Ruina, A. (1983). Slip instability and state variable friction laws. *Journal of Geophysical Research: Solid Earth*, 88, 10359–10370. <https://doi.org/10.1029/JB088iB12p10359>

7. Appendices

7.1 Appendix A1. Numerical stability of Euler–Cromer integration

To clarify why the Euler–Cromer method is used throughout this study, Figure A1 compares its behavior with that of the standard forward Euler scheme for a simple spring–mass oscillator. When integrated with the same time step, the forward Euler method exhibits systematic energy growth, causing the

oscillation amplitude to diverge unphysically over time. In contrast, the Euler–Cromer update keeps the total energy bounded and preserves the qualitative oscillatory motion. This stability property is essential for long stick–slip simulations, where artificial numerical energy drift would otherwise contaminate recurrence-time and slip-size statistics.

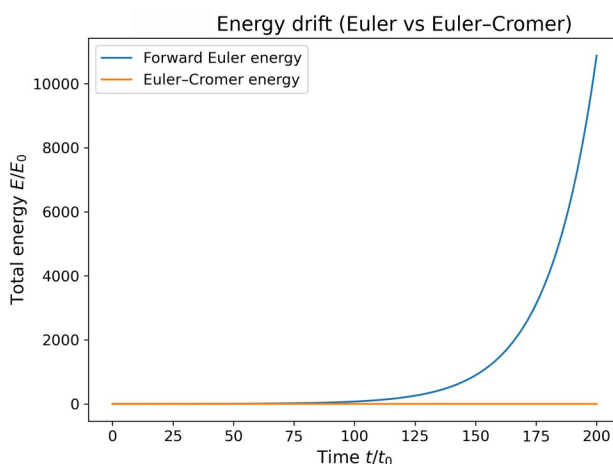


Figure A1. Energy behavior for a simple harmonic oscillator integrated by forward Euler versus Euler-Cromer (used in the BK simulations). Forward Euler exhibits energy drift, whereas Euler-Cromer keeps energy bounded. Axes: t/t_0 and E/E_0 .

7.2 Appendix A2. Limitations of cross-correlation lag as a synchronization metric
In the main text, synchronization between two blocks can be summarized by a time-lag statistic

derived from maximizing the displacement cross-correlation. For two displacement time series $x_1(t)$ and $x_2(t)$, define the normalized

cross-correlation and the “best lag” as $\rho(\Delta t) = \text{corr}(x_1(t), x_2(t + \Delta t))$, and $\tau = \arg \max_{\Delta t} \rho(\Delta t)$ respectively.

Figure A2 illustrates an important limitation of this lag-only diagnostic in a driven stick-slip system. Because both blocks share the same plate-loading trend and spend long intervals in near-stationary sticking, the cross-correlation can remain broad and peak near $\Delta t \approx 0$ even

when slip events are not truly coincident. In other words, a near-zero τ does not necessarily imply that the two blocks fail together; it can also reflect shared long-term loading and similar stick-phase plateaus. For this reason, the main text emphasizes an event-based synchronization measure (joint slip probability) based on event coincidence within a finite window w (here $w=1.0$), which directly targets the physical question “do they slip together?” rather than “are the long-term trends aligned?”

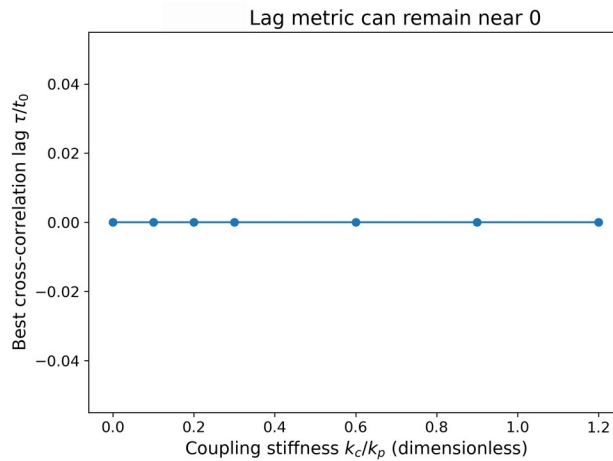


Figure A2. Best cross-correlation lag τ/t_0 between two-block displacement series can remain near 0 even when events are not strongly synchronized, motivating the event-coincidence metric in Figure 9.

7.3 Appendix A3. Loading-rate sensitivity

To broaden the parameter exploration beyond the baseline loading rate, we sweep the plate velocity V in the single-slider limit while holding ΔF fixed ($\Delta F=0.40$). Figure A3 plots

the mean recurrence $\langle T \rangle/t_0$ versus $1/V$, showing an approximately inverse dependence consistent with a simple drive-to-threshold estimate.

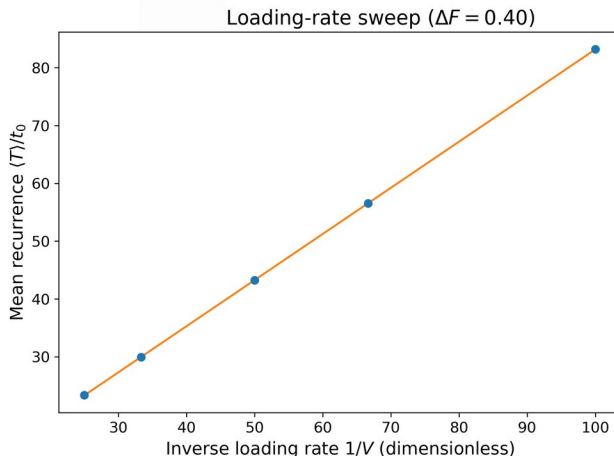


Figure A3. Loading-rate sensitivity (single-slider, $\Delta F = 0.40$): mean recurrence $\langle T \rangle / t_0$ as a function of inverse loading rate $1/V$. Points are means over multiple realizations (error bars ± 1 SE).

7.4 Appendix A4. System-size sensitivity

To assess finite-size effects, we compare mid-block recurrence estimates for chains of different lengths ($N = 25, 50$, and 100) using the baseline parameters. Figure A4 shows that the

mid-block mean waiting time $\langle T \rangle / t_0$ varies modestly with N over this range, supporting the use of $N = 50$ as a representative baseline while highlighting that system size is a controllable source of variability.

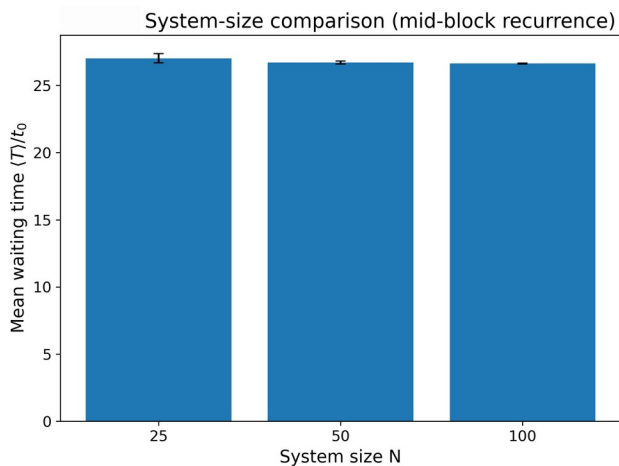


Figure A4. System-size sensitivity: mid-block recurrence estimates $\langle T \rangle / t_0$ for $N = 25, 50$, and 100 (baseline parameters). Bars are means over multiple realizations; error bars are ± 1 SE.

7.5 Appendix A5. Distribution-level summaries $\Delta x_{25}/x_0$ and local waiting times $\Delta t_{\text{start}}/t_0$, Several results in the main text are illustrated via representative time series. To provide a more systematic statistical view, Figure A5 plots complementary cumulative distribution functions (CCDFs) for the local slip-size proxy and size statistics are properties of an event population, not of a single run.

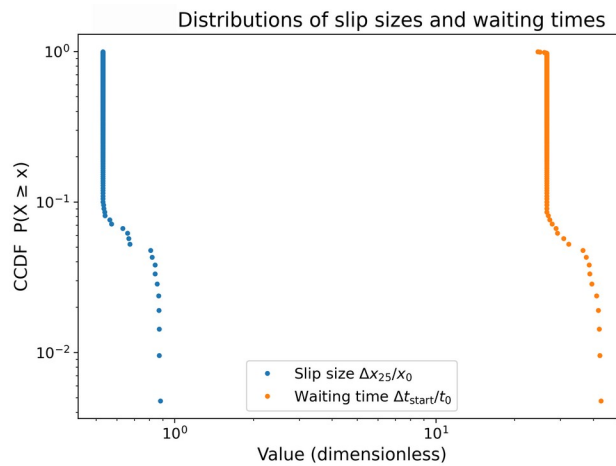


Figure A5. Complementary cumulative distribution functions (CCDFs) for local slip sizes $\Delta x_{25}/x_0$ and waiting times $\Delta t_{\text{start}}/t_0$ (block 25; pooled across multiple realizations).