Perspective

While the present analysis focuses on event statistics extracted under steady loading (Sections III–IV), the same event-detection framework can be extended to probe the system's response to controlled perturbations without introducing new observables. Specifically, using the existing velocity threshold vth, quiet-time merging tq, and participation criterion ϵ (Table 1), one can define a perturbation experiment in which a small displacement $\delta x \ll x0$ is applied to a single block at a time when the system is in the loading (stick) phase. The resulting response can be parsed using the same event-detection rules already employed throughout the manuscript, allowing direct comparison with baseline statistics.

Within this framework, a perturbation-triggered event can be characterized by its size:

$$S_{cascade} = \sum_{i=1}^{N} \left| \left(Delta\, x_i \right) \right|$$

and the corresponding participation fraction:

$$P = \frac{1}{N} \sum_{i=1}^{N} H \left(\left| \left(Delta\, x_i \right) \right| - epsilon \right)$$

(where H(·) is the Heaviside step function

The mean amplification is then defined as:

$$\chi = E \left(P \cdot N \right)$$

and represents the expected number of blocks activated by a localized perturbation, providing a natural extension of the event-size observable already reported in Fig. 10. Because the paper already demonstrates that event sizes and waiting times depend on coupling (Figs. 8–10) and detection thresholds (Fig. 11), this perturbation-based measure can be interpreted as probing how those same mechanisms control the propagation of disturbances rather than their spontaneous occurrence.

Importantly, this formulation allows direct comparison with existing results. For example, the degeneracy identified in Fig. 14—where distinct parameter sets yield identical mean recurrence ⟨T⟩ but different mean slip sizes—suggests that such cases may be distinguishable by their perturbation response, i.e., by differences in χ or in the distribution of $S_{cascade}$. Similarly, the location-dependent bias shown in Fig. 13 implies that perturbation outcomes may vary depending on where the disturbance is introduced, providing a natural extension of the spatial heterogeneity already documented. Finally, because the predictability horizon (Fig. 12) quantifies divergence under infinitesimal perturbations, the proposed finite-amplitude perturbation experiment can be viewed as a complementary, event-level analogue that connects trajectory divergence to observable cascade statistics.

The perturbation-response analysis does not introduce a new model but rather reuses the existing simulation and event-parsing pipeline to quantify how localized disturbances propagate through the same dynamics that generate the reported recurrence and synchronization behavior. This provides a concrete pathway to distinguish systems that are statistically similar under steady loading but differ in their susceptibility to cascading failure, strengthening the interpretation of the model in the context of earthquake triggering.

The proposed perturbation–response framework provides a natural bridge between the simplified spring–block dynamics studied here and real earthquake systems, where the key question is not merely when failure initiates but whether it propagates. In natural faults, small stress perturbations —arising from nearby earthquakes, fluid injection, or tidal forcing—can either dissipate locally or trigger cascading rupture, depending on elastic coupling and stress heterogeneity. This behavior is well documented in studies of earthquake triggering and stress transfer, particularly through Coulomb stress change analysis, which shows that modest stress perturbations on the order of tens of kilopascals can influence subsequent seismicity (Stein, 1999; King et al., 1994).

Within this context, the perturbation amplification metric introduced here (mean cascade size or effective number of triggered blocks, denoted χ) can be interpreted as a toy-model analogue of the branching ratio used in statistical seismology (Ogata, 1988; Helmstetter and Sornette, 2002). Systems with low effective branching exhibit predominantly isolated events, whereas systems approaching criticality display enhanced cascade potential and a higher likelihood of large, system-spanning ruptures. This distinction is reflected in real tectonic settings, such as the heterogeneous behavior of the San Andreas Fault compared to strongly coupled subduction zones in the Pacific Ring of Fire.

The framework also yields several testable quantitative predictions. First, the mean amplification χ is expected to increase monotonically with coupling stiffness k_c, with a transition from localized response ($\chi \approx 1$) to cascading behavior ($\chi \gg 1$) as the ratio k_c / k_p increases. Second, finite-size effects imply that χ should scale with system size N, with larger systems exhibiting higher maximum cascade sizes and a sharper transition between localized and global regimes; specifically, near a critical regime, χ may grow approximately as a power of N before saturating at values on the order of N. Third, the distribution of cascade sizes is predicted to broaden with increasing coupling, transitioning from narrow, exponentially decaying distributions in weakly coupled systems to heavy-tailed or near power-law behavior as the system approaches a critical state. Fourth, parameters that control loading and failure thresholds, such as the friction gap ΔF and loading rate V, are expected to modulate susceptibility indirectly. For example, larger ΔF (stronger effective barriers) should suppress cascade propagation and reduce χ, while higher loading rates may increase the likelihood that perturbations occur near failure and thus enhance cascade probability.

Framed in this way, the perturbation response provides a complementary observable to recurrence statistics: systems that appear similar when characterized by mean recurrence intervals may differ substantially in their susceptibility to cascading failure. This offers a potential explanation for why faults under comparable stress conditions can produce markedly different seismic outcomes, emphasizing the role of interaction-driven amplification rather than absolute stress level alone.

Limitations

Several limitations should be noted when interpreting these results in the context of real earthquake systems. The friction law employed here is a simplified static–kinetic switch and does not capture rate-and-state dynamics, which are known to control nucleation size and rupture propagation (Dieterich, 1979; Ruina, 1983). Consequently, the predicted scaling relationships for perturbation amplification should be interpreted as qualitative trends rather than quantitatively calibrated laws. In addition, the model assumes homogeneous parameters and one-dimensional geometry, whereas real faults exhibit spatial heterogeneity, complex geometries, and multi-scale interactions.

Future work

Future work should therefore focus on testing the robustness of the predicted scaling relationships—particularly the dependence of χ on coupling strength, system size, and frictional parameters—in more realistic settings. Incorporating rate-and-state friction would allow direct comparison with theoretical nucleation length scales, while extending the model to heterogeneous or higher-dimensional systems could reveal whether the predicted transition in cascade behavior sharpens into a true critical phenomenon. A particularly important direction is to compare model-derived cascade statistics with empirical earthquake triggering data, such as aftershock productivity and inferred branching ratios from seismic catalogs. Such comparisons would help determine whether susceptibility to perturbations can serve as a quantitative diagnostic for distinguishing between fault systems that are prone to localized slip versus those capable of large, cascading earthquakes.

Quantitative predictions

- **Coupling scaling:**
  χ increases with kc/kp; expect a crossover from $\chi \approx 1 \rightarrow \chi \sim O(N)$
- **System-size scaling:**
  Near criticality:
  $\chi \propto N^\alpha$ $(0 < \alpha \leq 1)$, saturating at $\chi \sim N$
- **Cascade-size distribution:**
  Weak coupling → exponential decay
  Strong coupling → heavy-tailed / near power-law
- **Friction gap dependence:**
  Increasing ΔF → decreases χ (harder to propagate slip)
- **Loading-rate dependence:**
  Increasing V → increases probability of large cascades (system closer to threshold when perturbed)

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