Peer-Review

Yin, Benjamin, and Noah Aydin. 2026. “Using Impartial Games and Greedy Algorithms to Generate Codes with Fixed Distance.” *Journal of High School Science* 10 (1): 37–59. https://doi.org/10.64336/001c.155366

1.The paper connects B–greedy codes and impartial games pedagogically and sketches a weak GV proof via greedy codes. To strengthen novelty, please (1) provide complete, rigorous proofs with precise hypotheses; (2) include computational generation/verification of lexicodes and B–orders with code/data; (3) compare dimensions/rates of constructed codes against best-known bounds; (4) clarify the exact new contributions versus implications of Brualdi–Pless; (5) extend beyond binary to q-ary settings if possible.
2.The paper assumes equivalence between g-values assigned via a greedy ordering and Sprague–Grundy values under particular ‘turning set’ rules; these require more precise statement and justification. Several theorems depend on homomorphism properties taken from prior work without detailed conditions.
3.Results are mainly theoretical with small illustrative examples; please systematically explore, quantitate and discuss sensitivity to changing orderings, distances d, or game variants. Provide and discuss computational experiments or counterexamples.
4.Many arguments are plausible and align with known results (e.g., linearity of lexicodes, homomorphism mapping to syndromes), but several proofs are sketches, omit key steps, or circularly reference later results (e.g., using a corollary not yet established).
5.There is inconsistent terminology around P-positions vs ‘winning positions’ for the second player; a misdated reference; and several proof sketches rely on unproven or deferred statements. Some claims of novelty overlap with known results. Examples align with stated rules but are limited in breadth. There is dependence on results proved later in the text.
6.The paper does not engage with open performance gaps (e.g., optimality vs. SOTA) or recent literature. Please address in the manucript.
7.I do not understand how the use of greedy algorithms to prove (a version of) the Gilbert-Varshamov bound is novel. This has been done before—in fact, it is the foundation of the original proof; i.e. starting with an empty code, iteratively add vectors that are at distance $\geq d$ from existing ones. This greedy construction is the core idea behind the GV bound. Please therefore explicitly explain and discuss in the manuscript what exactly is novel or new about your proof.
8.Multiple grammatical errors, typos (e.g., ‘lexigraphic’, ‘my choose’), inconsistent notation, and tense/register not consistently in formal scientific style. Several sentences are awkward or ambiguous. At least one citation is incorrect; i.e. appears misdated (Conway & Sloane paper listed as 2003 though the DOI corresponds to 1986). Please thoroughly check citations.
9.Please submit a word file formatted to the Journal’s guidelines in addition to a pdf file.

To:
Shireesh Apte, Ph.D., Editor
Journal of High School Science

27 October 2025

Dear Dr. Apte,

Together with Prof. Aydin, we would like to thank you for your detailed review of our manuscript “**Using Impartial Games and Greedy Algorithms to Generate Codes with Fixed Distance**” and the notification that you will consider a revised version that addresses the concerns of the reviewer.

We have carefully considered the comments and have addressed each one in point-by-point responses, which also describe any associated revisions made in the manuscript. We submitted 4 copies of our manuscript, 2 are in pdf form and 2 are in word form. One version of the manuscript has highlights representing the changes we made and the other has these highlights removed to prepare it for publication. We thank you and the reviewer for your interest in the data and the helpful comments. We believe that the manuscript has been strengthened by the revision process and hope that you will now find it acceptable for publication.

Please do not hesitate to contact us if you have any queries. Thank you again for your consideration of this revised manuscript.

Sincerely,

Benjamin Yin
Lexington High School, 251 Waltham St, Lexington, MA 02421, USA.

Responses to comments:

Reviewer #1:

1. The paper connects B–greedy codes and impartial games pedagogically and sketches a weak GV proof via greedy codes. To strengthen novelty, please (1) provide complete, rigorous proofs with precise hypotheses; (2) include computational generation/verification of lexicodes and B–orders with code/data; (3) compare dimensions/rates of constructed codes against best-known bounds; (4) clarify the exact new contributions versus implications of Brualdi–Pless; (5) extend beyond binary to q-ary settings if possible.

Author response: (1) The authors have revised the proofs to be rigorous and complete. (2) The authors added appendix A and appendix B, showcasing two original python codes created to calculate B-orders and lexicodes. (3) The authors have added appendix C, which compares dimensions of lexicodes to that of best-known bounds. (4) The authors clarified that the exact new contributions versus implications of Brualdi-Pless were the three results (some of which may be deduced by Brualdi-Pless's work, but we present a proof for it that was not included in their work) Lemma 4.1, Lemma 4.2, and Theorem 4.3. (5) The authors have directed readers to a recent paper on q-ary lexicodes.

Page(s) and section(s) for changes made: Appendix A and Appendix B on page 16.

2. The paper assumes equivalence between g-values assigned via a greedy ordering and Sprague–Grundy values under particular 'turning set' rules; these require more precise statement and justification. Several theorems depend on homomorphism properties taken from prior work without detailed conditions.

Author response: The authors have edited the theorem relating g-values with G-values (Theorem 4.2) to a more precise statement. The justification has also been edited to add clarity. The theorems that depend on homomorphism properties are now put after Theorem 4.1 which introduces the idea of a homomorphism. All proofs involving homomorphisms have detailed conditions now and are based on standard properties of homomorphisms which are not necessary to include in this manuscript.

Page(s) and section(s) for changes made: Section 4.5 on page 10.

3. Results are mainly theoretical with small illustrative examples; please systematically explore, quantitate and discuss sensitivity to changing orderings, distances d, or game variants. Provide and discuss computational experiments or counterexamples.

Author response: The authors added comments in the manuscript related to changing orderings and how slightly changing orderings does not affect B-greedy codes. The authors have also included a comment about how slight changes in distance d does not have a major effect on the change in dimension. The authors provided computational experiments relating to changing distance d, and data on distance and dimension can be found in Appendix C.

Page(s) and section(s) for changes made: Section 4.1 on Page 9 and Section 4.4 on Page 10.

4. Many arguments are plausible and align with known results (e.g., linearity of lexicodes, homomorphism mapping to syndromes), but several proofs are sketches, omit key steps, or circularly reference later results (e.g., using a corollary not yet established).

Author response: The authors have changed the order of results so that the authors begin section 4 by introducing B-greedy codes and then presenting Brualdi-Pless' key theorem along with the corollary. This way, the readers will know about the results needed to understand the section on lexicodes. The authors have marked the beginning and end of the moved section in orange in the manuscript. Proofs have been edited to not omit key steps or circularly refer to later results.

Page(s) and section(s) for changes made: Section 4.1 on Page 9.

5. There is inconsistent terminology around P-positions vs 'winning positions' for the second player; a misdated reference; and several proof sketches rely on unproven or deferred statements. Some claims of novelty overlap with known results. Examples align with stated rules but are limited in breadth. There is dependence on results proved later in the text.

Author response: The inconsistent terminology around P-positions has been deleted from the manuscript. Proofs have now been rearranged so that proofs do not rely on unproven or deferred statements. Claims of novelty have been edited, and the authors understand that they overlap with known results and have edited the statements to be more accurate (e.g. Gilbert-Varshamov is not newly discovered but appears when exploring B-greedy codes). Examples are no longer limited in breadth (The authors added an extra term to the example on generating B-orders). There is no longer dependence on results proved later in the text as the authors have moved Theorem 4.1 and Corollary 4.1 to the beginning of section 4. The misdated reference has been changed.

Page(s) and section(s) for changes made: References, Section 4.1 on Page 9, Section 5.6 on Page 14.

6. The paper does not engage with open performance gaps (e.g., optimality vs. SOTA) or recent literature. Please address in the manuscript.

Author response: The manuscript now engages with recent literature. The authors referenced Bogdanova and Todorov's paper from 2022.

Page(s) and section(s) for changes made: Section 4.4 on Page 10.

7. I do not understand how the use of greedy algorithms to prove (a version of) the Gilbert-Varshamov bound is novel. This has been done before—in fact, it is the foundation of the original proof; i.e. starting with an empty code, iteratively add vectors that are at distance $\geq d$ from existing

ones. This greedy construction is the core idea behind the GV bound. Please therefore explicitly explain and discuss in the manuscript what exactly is novel or new about your proof.

Author response: The authors agree that the Gilbert-Varshamov bound is not novel. The authors have edited the manuscript to address these concerns of novelty.

Page(s) and section(s) for changes made: Section 5.6 on Page 14.

8. Multiple grammatical errors, typos (e.g., 'lexigraphic', 'my choose'), inconsistent notation, and tense/register not consistently in formal scientific style. Several sentences are awkward or ambiguous. At least one citation is incorrect; i.e. appears misdated (Conway & Sloane paper listed as 2003 though the DOI corresponds to 1986). Please thoroughly check citations.

Author response: Grammatical errors, typos, inconsistent notation, and tense have been edited to adhere to formal scientific style. The authors have edited sentences that may sound awkward or ambiguous. Conway & Sloane paper citation has been edited from 2003 to the correct year 1986.

Page(s) and section(s) for changes made: Whole manuscript and references.

9. Please submit a word file formatted to the Journal's guidelines in addition to a pdf file.

Author response: The authors have submitted a word file and a pdf file that adhere to the Journal's guidelines. The authors have also included two copies of the manuscript, one with highlights to indicate edits and the other without highlights.

Page(s) and section(s) for changes made: Whole manuscript.

Thank you for addressing my comments. A few inconsistencies remain.
1.Factual errors: Core definitions and classical results are accurate, but Theorem 5.4 (all maximal linear codes are B-greedy codes) appears insufficiently justified: the proof conflates linear maximality with non-extendability under the B-greedy selection and does not rule out non-linear extensions or guarantee g-value assignments prevent further zero labels; this is a potential gap. Similarly, several proofs would benefit from several proofs would benefit from separation of linear versus non-linear cases. Please address in the manuscript.
2.The paper shows some order-invariance properties (e.g., L_n ⊕ w) and links to homomorphisms, but it lacks a broader sensitivity analysis over bases, thresholds d, and q-ary alphabets, and does not test robustness of derived dimensions across many instances. I suggest adding a complexity analysis, and explore q-ary generalizations and alternative orderings beyond L_n ⊕ w with systematic experiments.
3.Please upload a word file as a one-column document, 12 times new roman font. Please see formatting guidelines for the Journal.

To:
Shireesh Apte, Ph.D., Editor
Journal of High School Science

30 October 2025

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We have carefully considered the comments and have addressed each one in point-by-point responses, which also describe any associated revisions made in the manuscript. We submitted 4 copies of our manuscript, 2 are in pdf form and 2 are in word form. One version of the manuscript has highlights representing the changes we made and the other has these highlights removed to prepare it for publication. We thank you and the reviewer for your interest in the data and the helpful comments. We believe that the manuscript has been strengthened by the revision process and hope that you will now find it acceptable for publication.

Please do not hesitate to contact us if you have any queries. Thank you again for your consideration of this revised manuscript.

Sincerely,

Benjamin Yin
Lexington High School, 251 Waltham St, Lexington, MA 02421, USA.

Responses to comments:

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Author response: The authors have edited the proof of Theorem 5.4 to rule out the possibility of a non-linear extension, which implies that no further g-value can be assigned a zero label. This manuscript is only concerned with linear codes resulting from greedy algorithms so the separation of linear versus non-linear cases feels unnecessary. The authors believe that the focus on linear cases is implied in the manuscript, and we will work on non-linear cases for future manuscripts but not this one.

Page(s) and section(s) for changes made: Section 5.6 Theorem 5.4 on Page 19

2. The paper shows some order-invariance properties (e.g., L_n ⊕ w) and links to homomorphisms, but it lacks a broader sensitivity analysis over bases, thresholds d, and q-ary alphabets, and does not test robustness of derived dimensions across many instances. I suggest adding a complexity analysis, and explore q-ary generalizations and alternative orderings beyond L_n ⊕ w with systematic experiments.

Author response: The authors appreciate the reviewer's suggestion for generalizations upon the results presented in this manuscript. However, for this manuscript, we would like to limit our scope to the simple cases of lexicodes and B-greedy codes resulting from B-orderings or L_n ⊕ w. We will

work on exploring extensions in later work, but for this manuscript we would appreciate it if you could publish it with our current results.

3. Please upload a word file as a one-column document, 12 times new roman font. Please see formatting guidelines for the Journal.

Author response: The manuscript in pdf form should be font 12 times new roman with math formulas in math mode. Upon converting to a word file, plaintext is still font 12 times new roman, however, word is not able to compile a lot of the math formulas we have written using latex. For publication, we would suggest using the pdf format. The manuscript is now a one-column document.

Thank you for addressing my comments. I have two suggestions before I can approve this manuscript.
1.Please include this verbage (or equivalent) at an appropriate location in the manuscript if you have not already done so "Proof 5.4 rules out the possibility of a non-linear extension, which implies that no further g-value can be assigned a zero label. This work is only concerned with linear codes resulting from greedy algorithms hence the separation of linear versus non-linear cases is not deemed necessary."
2.Similarly, please include this verbage (or equivalent) at an appropriate location in the manuscript. "In this work, we have limited our scope to the simple cases of lexicodes and B-greedy codes resulting from B-orderings or L_n ⊕w. Therefore, a broader sensitivity analysis over bases, thresholds d, and q-ary alphabets is not attempted, and neither is the robustness of derived dimensions across many instances assessed."

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Author response: The authors appreciate the reviewer's suggestion for generalizations upon the results presented in this manuscript. However, for this manuscript, we would like to limit our scope to the simple cases of lexicodes and B-greedy codes resulting from B-orderings or L_n ⊕ w. We will work on exploring extensions in later work, but for this manuscript we would appreciate it if you could publish it with our current results.

3.Please upload a word file as a one-column document, 12 times new roman font. Please see formatting guidelines for the Journal.
Author response: The manuscript in pdf form should be font 12 times new roman with math formulas in math mode. Upon converting to a word file, plaintext is still font 12 times new roman, however, word is not able to compile a lot of the math formulas we have written using latex. For publication, we would suggest using the pdf format. The manuscript is now a one-column document.

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Journal of High School Science

7 Nov 2025

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Please do not hesitate to contact us if you have any queries. Thank you again for your consideration of this revised manuscript.

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Author response: We have included this verbage immediately after the proof of Theorem 5.4.

Page(s) and section(s) for changes made: Section 5.6 on Page 19

2. Similarly, please include this verbage (or equivalent) at an appropriate location in the manuscript. “In this work, we have limited our scope to the simple cases of lexicodes and B-greedy codes resulting from B-orderings or L_n ⊕w. Therefore, a broader sensitivity analysis over bases, thresholds d, and q-ary alphabets is not attempted, and neither is the robustness of derived dimensions across many instances assessed.”

Author response: We have also included this verbage, located near the end of the conclusion.

Page(s) and section(s) for changes made: Conclusion on Page 21

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Thank you for addressing my comments. Accepted.