



Is Cultural or civilizational exchange, diffusion, or contact a requirement for the formation and evolution of music?

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Abstract

Eastern music, often characterized by its use of microtones, complex rhythms, and unique melodic systems, contrasts with the Western tradition, which emphasizes harmonic structures, tonality, and fixed pitches. Despite these differences, the historical encounters between Eastern and Western musical cultures have produced intriguing crossovers. Notable examples include the integration of Indian and Persian musical ideas in the early 20th century through the works of composers like Claude Debussy and Ravi Shankar's collaboration with George Harrison of The Beatles. This raises the critical question of whether cultural exchange is a prerequisite for these musical fusions, or if similar principles of music could have naturally emerged in both cultures.. The use of tools such as mathematical analysis, including cosine similarity and Fourier transform, can offer new perspectives on the similarities and differences between musical traditions, providing insight into whether these shared traits arise naturally and/or independently or are the result of long-term cultural contact.

Keywords

Music, Eastern music, Western music, Music convergence, Cross-cultural music, Fusion music, Merging music, Ethnomusicology, Fourier transform, Cultural diffusion

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Introduction

Cultural diffusion refers to the process by which cultural elements—such as ideas, practices, languages, or artistic traditions—are spread from one society to another. Historically, cultural diffusion has been a significant force in shaping the development of human societies, leading to the blending of practices and the creation of hybrid forms of art, literature, language, and music. In the context of music, cultural diffusion plays a crucial role in the exchange of musical techniques, instruments, and even entire genres across different civilizations. This paper explores how such exchanges have influenced the development of musical traditions and asks whether they are necessary preconditions for musical merging or whether intrinsic musical principles could lead to similar developments across cultures. The interaction between Eastern and Western music traditions is an exemplary case of cultural diffusion, offering a rich tapestry of influences, collaborations, and innovations.

Throughout history, music from the East and West has been influenced by numerous factors such as trade, migration, and colonization. For instance, the Silk Road, an ancient trade route linking the East and West, facilitated the exchange not only of goods but also of cultural and musical ideas. Western classical music, with its roots in European religious and folk traditions, encountered Eastern musical systems—such as Indian ragas or Chinese pentatonic scales—during these exchanges, leading to a blending of harmonic, rhythmic, and instrumental styles.

Eastern music, often characterized by its use of microtones, complex rhythms, and unique melodic systems, contrasts with the Western tradition, which emphasizes harmonic structures, tonality, and fixed pitches. Despite these differences, the historical encounters between Eastern and Western musical cultures have produced intriguing crossovers. Notable examples include the integration of Indian and Persian musical ideas in the early 20th century through the works of composers like Claude Debussy and Ravi Shankar's collaboration with George Harrison of The Beatles.

Discussion

This raises the critical question of whether cultural exchange is a prerequisite for these musical fusions, or if similar principles of music could have naturally emerged in both cultures. The use of tools such as mathematical analysis, including cosine similarity, can offer new perspectives on the similarities and differences between musical traditions, providing insight into whether these shared traits arise naturally or are the result of long-term cultural contact (see appendix).

Carnatic music, one of the two main traditions of Indian classical music (the other being Hindustani music), has a profound historical and cultural significance, particularly in southern India. Carnatic music is built around ragas (melodic structures) and talas (rhythmic cycles), emphasizing improvisation, ornamentation, and emotional expression. Its roots date back over 2,000 years, with its formal system taking shape in the 15th to 17th centuries. The greatest composers of all time like Tyagaraja, Muthuswami Dikshitar, and

Shyama Shastri are credited with developing its core repertoire of devotional music, deeply intertwined with Hindu religious rituals and philosophies.

Unlike Western music, which historically developed around harmony, melody, and orchestral arrangements, Carnatic music is centered on the melodic and rhythmic aspects of performance, with a particular focus on the voice and solo instruments like the veena, mridangam, and violin. The interaction between raga and tala forms the basis of a performance, where the performer improvises within these strict frameworks, allowing for both spontaneity and structure. This deep connection between music, spirituality, and cultural identity has made Carnatic music a key element of southern Indian heritage, often performed in temples and during festivals as a form of devotion.

On the other hand, Western music has followed a different evolutionary path, beginning with ancient Greek music theory and growing through Roman, medieval, Renaissance, Baroque, Classical, Romantic, and modern periods. Its system of harmony, major/minor tonality, and orchestration of instruments such as the piano, strings, and brass has defined much of Western classical music. Over time, Western music developed an intricate system of notation that enabled composers to write down and share their work, leading to the formation of symphonies, operas, and concertos that became the foundation of Western classical traditions.

The historical significance of both Carnatic and Western music traditions lies in their development as distinct yet sophisticated systems that reflect the unique cultural values and social structures of their respective societies. While Carnatic music emphasizes emotional depth, improvisational skill, and spiritual devotion, Western music has tended to prioritize structural complexity, harmonic development, and orchestral balance. However, despite these differences, both traditions have influenced each other through ancient and medieval trading routes, colonial interactions, modern globalization, and cross-cultural collaborations.

The integration of Western instruments such as the violin into Carnatic music during the colonial period is one such example of cultural exchange. Composers like Muthuswami Dikshitar incorporated Western harmonic ideas into their compositions, resulting in a fusion of the two traditions. These exchanges have become more pronounced in the 20th and 21st centuries, with Indian musicians embracing Western music theory and instruments, while Western musicians have drawn inspiration from Indian ragas and rhythms.

In exploring the historical significance of these musical traditions, it is apparent that while they developed separately, the growing exchange between them, especially in the modern era, highlights the interconnectedness of human creativity and the shared potential for cross-cultural influence. Thus, the merging of musical traditions may not only stem from direct cultural diffusion but could also arise

from parallel creative impulses across different societies.

While cultural exchange and diffusion often facilitate the merging of musical styles, this study applies mathematical methods such as cosine similarity and Fourier transform to provide an empirical framework for analyzing the shared musical features and understanding the extent of cross-cultural influences (see appendix).

The concept of cultural diffusion in music

Cultural diffusion refers to the spread of cultural elements—such as beliefs, practices, languages, technologies, and artistic forms—across different societies. This process often occurs through various channels, including trade, migration, warfare, colonization, and more recently, globalization. As people move and interact, they exchange ideas and practices, leading to the adoption, adaptation, and transformation of cultural elements.

In the context of the arts, particularly music, cultural diffusion plays a pivotal role in the evolution and diversification of musical traditions. It facilitates the introduction of new instruments, scales, rhythmic patterns, and performance styles to different regions, allowing for the creation of hybrid musical forms and the enrichment of existing ones. For example, the influence of African rhythms on Western jazz or the integration of Western instruments into Indian classical music demonstrates how music can evolve through the exchange of cultural elements.

Throughout history, major trade routes like the Silk Road, the spread of religions such as Islam, and the colonial expansion of European powers have all contributed to significant cultural diffusion. For instance, during the period of the Mughal Empire in India, the fusion of Persian musical traditions with indigenous Indian styles led to the development of new forms in Indian classical music. Similarly, in the 19th and 20th centuries, Western classical music encountered Indian classical music through cross-cultural interactions, particularly in the context of colonial rule, which influenced composers in both traditions.

Cultural diffusion also plays a role in the spread of musical technology, such as the adoption of Western musical instruments in various regions, or the exchange of musical notation systems, which helped preserve and disseminate musical knowledge across the world.

Additionally, the globalization of media in the 20th and 21st centuries has facilitated the spread of music across borders more rapidly, leading to even greater cross-cultural influences and the emergence of global music genres. In sum, cultural diffusion acts as a catalyst for innovation and transformation in music, fostering both the preservation of traditional forms and the creation of new, hybrid musical expressions. By examining how musical traditions interact and merge, we can better understand the dynamic ways in which cultural exchange shapes artistic identities globally.

Cultural diffusion has played a pivotal role in shaping global musical traditions by facilitating the exchange of ideas, instruments, and techniques across cultures. One of the most significant examples is the Silk Road, an ancient network of trade routes that connected the East and West, where music from Persia, Central Asia, and China influenced one another. For example, instruments such as the pipa from China and the sitar from India spread along these routes, and Persian and Arabic musical ideas left a lasting imprint on Central Asian and Chinese music. The Mongol Empire further accelerated this diffusion by blending musical practices from various regions, such as Persia, China, and Eastern Europe.

Similarly, during the colonial era, Western influence profoundly impacted Indian classical music. Western instruments like the violin were integrated into Carnatic music ensembles, and composers such as Muthuswami Dikshitar incorporated Western harmonic ideas into Indian classical compositions. This fusion helped shape a new sound while still maintaining the integrity of traditional forms.

Another notable example of cultural diffusion in music is the African diaspora, particularly during the transatlantic slave trade, which led to the merging of African musical elements with European traditions in the Americas. African rhythms and polyrhythms deeply influenced the creation of jazz, blues, and rock music, with African-American artists like Louis Armstrong and Duke Ellington combining African and European musical practices to produce jazz, a genre that eventually became a global phenomenon. Likewise, Caribbean

musical traditions, including calypso, reggae, and ska, significantly influenced the development of Western popular music, with figures like Bob Marley bringing reggae to international prominence and shaping genres like funk and hip-hop. The Islamic influence on Spain during the period of Muslim rule in the Iberian Peninsula (8th-15th centuries) also stands as a key moment in musical diffusion, the Fado and the Flamenco being prominent.

The introduction of Arabic instruments like the oud and qanun to Spain contributed to the evolution of the lute in European music. This blending of Arabic and Christian musical traditions in Spain led to the development of new forms such as flamenco, which incorporated both Moorish and Gypsy influences. In the 19th and 20th centuries, Western classical music began to influence Chinese music, particularly with the introduction of Western instruments and musical theory. Chinese composers like Tan Dun and Zhou Long started integrating traditional Chinese music with Western styles, creating a fusion that merged Eastern and Western musical elements. These historical moments highlight how cultural diffusion has been a driving force in the evolution of music, blending different traditions and enriching musical expressions around the world.

The question of whether cultural contact is necessary for the emergence of musical similarities or if parallels can naturally arise due to universal aspects of human creativity is a complex one. While historical examples of cultural diffusion strongly suggest that direct contact often leads to the creation of shared

musical elements, the question remains whether certain features of music might emerge independently due to cognitive universals or environmental factors. On one hand, historical evidence of cultural diffusion - through trade, migration, colonization, and globalization - demonstrates that interaction between different societies often leads to the exchange of ideas, techniques, and artistic practices, including in music. However, on the other hand, the similarities between various musical traditions that arise independently of cultural contact suggest that human creativity might have inherent tendencies toward certain forms of expression.

Cultural contact has undoubtedly played a crucial role in the development of music. The Silk Road, colonialism, and the African diaspora are just a few examples where musical traditions were influenced and transformed through interaction between different cultures. For instance, the incorporation of Western instruments like the violin into Carnatic music and the blending of African rhythms with Western jazz clearly resulted from cross-cultural exchanges. These moments of contact provided fertile ground for the blending of musical ideas, resulting in new forms of music that would not have been possible without such interaction.

However, there is also evidence to suggest that musical parallels can arise independently across cultures due to universal aspects of human creativity. One compelling argument for this is the recurring appearance of similar musical features, such as the pentatonic scale, in cultures that had no direct contact. For

example, many cultures have independently developed scales and systems based on the division of the octave, though the exact intervals and the ways in which music is structured may differ. The use of rhythm and melody as tools for emotional expression is another example of a universal trait found in nearly every culture, even in the absence of direct contact. The existence of similar musical features—such as pentatonic scales in both Eastern and Western music—may arise from shared human cognitive abilities, such as the perception of certain intervals as consonant or pleasing to the ear. This suggests that while cultural diffusion can explain many cross-cultural musical similarities, there may also be fundamental biological; either evolutionally conserved or evolutionally limiting; aspects of human cognition and creativity that lead to similar musical outcomes, even without direct interaction.

In addition, the theory of parallel development suggests that societies with similar environmental and social conditions may develop analogous artistic forms. These universal conditions, including the need for rhythm in social rituals or the expression of emotional experiences, may have prompted the independent emergence of comparable musical structures in various cultures. For instance, the need for rhythm in community rituals, labor, or dance could lead to the independent development of rhythm-based musical structures in different parts of the world. Similarly, the desire to convey emotion or tell stories through music might lead to the independent development of vocal and

instrumental traditions aimed at expressing human experience.

Ultimately, cultural contact has undoubtedly shaped much of the diversity and innovation in global music. However, the observation that similar musical ideas may emerge independently across different societies suggests that some elements of music might be a product of universal cognitive patterns, possibly emerging even in the absence of contact. The coexistence of both cultural influence and innate creativity highlights the dynamic ways in which music evolves—both through interaction and as a reflection of universal human impulses.

Cosine similarity in music analysis

Cosine similarity is a widely used mathematical measure for quantifying the similarity between two sets of data. It is particularly useful when comparing vectors in a multi-dimensional space, such as text, sound, or other forms of data. The cosine similarity between two vectors is calculated as a ratio of their dot product to the product of their magnitudes. Returned results range from -1 (the vectors point in the opposite direction, not similar) to 0 (the vectors are orthogonal, not similar) to 1 (the vectors point in the same direction, similar).

First, the Musical characteristics of each song are identified. These characteristics can include key, tempo, mode, pitch, energy, amplitude, danceability; among others. These characteristics are called features. Each individual feature value is part of a matrix; which itself can be interpreted as a point

(vector) in a multi-dimensional space, where each dimension corresponds to a specific feature. The two matrices (for the two songs or compositions) are then compared using the cosine similarity equation. By representing compositions as numerical vectors—where each dimension corresponds to a specific feature or aspect of the music—cosine similarity quantifies the degree of similarity between two pieces of music based on their features.

For example, when comparing Western and Eastern music, the cosine similarity measure could be applied to compositions from similar historical periods in both traditions. The vectors representing these compositions might capture data related to scales, intervals, rhythmic patterns, and other elements. By calculating the cosine similarity between these vectors, the method can assess whether the musical elements of Eastern and Western pieces converge, diverge, or remain distinct over time.

This approach allows researchers to quantify the degree of similarity between two compositions without needing to rely on subjective interpretation or human judgment. It provides a precise, data-driven method for analyzing musical structures across different genres or time periods, making it particularly useful for comparing diverse musical traditions such as Eastern and Western music.

To analyze differences between compositions from similar historical periods in Eastern and Western music using cosine similarity, we would begin by selecting representative

compositions from both traditions within a comparable historical timeframe, such as the Baroque period in Western music and the early modern periods in Eastern music (e.g., Mughal-era Indian classical music or Qing dynasty Chinese music). A set of influential compositions from both traditions would be chosen, such as works by J.S. Bach or Handel from the West and figures like Muthuswami Dikshitar or Zhi-Wei Zeng from the East. Next, we would extract relevant musical features from each composition, including pitch (e.g., scale or key), rhythm (e.g., tempo, note duration, or rhythmic cycles like tala in Indian music), harmony (e.g., chord progressions in Western music and drones in Eastern music), melodic contour, and instrumental arrangement (e.g., Western strings vs. Eastern instruments like the sitar or veena). These features would be translated into numerical vectors, where each dimension represents one of the musical elements. After constructing the vectors, cosine similarity would be applied to measure the degree of similarity between each pair of compositions—one from the Eastern tradition and one from the Western tradition. This calculation would be done by determining the cosine of the angle between the two vectors, where a higher score (closer to 1) indicates a greater similarity. The results would be organized into a similarity matrix, allowing for a detailed comparison of how Eastern and Western compositions align or differ based on their musical elements. This approach provides a systematic and quantitative method for analyzing cross-cultural musical comparisons.

Various other interpretations of this basic cosine similarity algorithm have been reported

in the literature. Georges (1) communicated the application of cosine similarity in comparing musical compositions. However, in this study, the cosine similarity was itself based on a subjective interpretation of musical influences (personal musical influences). Short Time Fourier Transform (STFT) has been used to identify the fundamental frequencies of notes in a melody, the constituent notes of a chord or as a genre classification tool (2-4).

In conclusion, cosine similarity offers a robust mathematical tool for comparing musical compositions, providing a way to objectively measure their similarities and differences. This technique can be used to explore the relationships between different musical traditions and track how musical elements evolve over time.

Case studies and historical trends

To analyze the convergence, divergence, or distinctness of musical styles between Eastern and Western traditions, we can select overlapping historical periods that allow for meaningful comparisons. For example, the Baroque period (1600-1750) in Western music corresponds with significant periods in both Indian classical music and Chinese classical music.

In Indian classical music, the Mughal period (16th to 19th centuries) saw the rise of intricate Hindustani and Carnatic music, with the formalization of Hindustani classical music under influential figures like Tansen. The Mughal period also saw the introduction of Persian influences into Indian music, particularly in the realms of melody and

rhythm, which laid the groundwork for further fusion with Western traditions. Similarly, Chinese classical music during the Qing dynasty (1644-1912) reflects a time of flourishing musical development, especially in compositions for traditional instruments like the guqin and pipa. For the Western Baroque tradition, composers such as Johann Sebastian Bach, George Frideric Handel, and Antonio Vivaldi exemplified the contrapuntal textures, harmonic experimentation, and ornamentation characteristic of this period. Although Western music was influenced by its religious and cultural context, it did not experience the same degree of cross-cultural fusion as Eastern music during this period.

To compare these traditions, we would represent the musical features of selected compositions from each tradition as vectors, capturing key elements such as pitch (scales and intervals), rhythm (time signatures and tala cycles), harmony (chord progressions, drones, or harmonic relations), and melodic contour. In this process, we would focus on how the fundamental structures of music in each tradition—whether tonal or modal—create different sensory experiences. Western Baroque music, with its emphasis on tonality and harmonic progressions, can be contrasted with Eastern traditions, which often focus on modal systems and drone-based structures. The cosine similarity between these vectors would allow us to measure how similar or different the musical styles are, highlighting whether these traditions converged, diverged, or remained distinct over time.

The influence of historical events such as colonization, globalization, and other socio-political changes has significantly impacted the convergence, divergence, or distinctness of Eastern and Western musical traditions. For instance, during the colonial period, Western powers established control over vast parts of Asia and Africa, which led to the imposition of Western cultural norms, including musical forms. Colonial powers, particularly the British, saw Western classical music as a tool for cultural dominance, incorporating it into educational institutions and establishing Western orchestras in India. In India, British colonization profoundly affected both Hindustani and Carnatic music. The British introduced Western classical music through educational institutions and cultural exchanges, leading to a blending of Western and Indian musical traditions. This fusion can be seen in the incorporation of Western instruments, such as the violin, into Indian classical ensembles, which resulted in a hybridized sound, particularly in Carnatic music. Similarly, during the British Raj, Western harmonies and orchestration methods began to influence Indian composers like Muthuswami Dikshitar, who incorporated Western ideas into his compositions while still maintaining traditional Indian frameworks. This period marked a transformative moment in musical history, where Eastern and Western traditions began to intermingle, creating a cross-pollination of musical styles.

Technological progress further intensified the exchange of musical ideas, particularly in the 20th century. The advent of radio, television, and later, the internet, facilitated the

dissemination of Western music worldwide, exposing Eastern audiences to genres like classical, jazz, and rock. The ability to distribute music through digital platforms allowed Eastern audiences to become aware of Western genres that were previously inaccessible, broadening the scope for musical hybridization. Conversely, Eastern music, particularly Indian and Chinese classical traditions, gained popularity in the West, with artists such as Ravi Shankar bringing sitar music into Western consciousness. This cross-cultural exchange led to new hybrid genres, like fusion music, blending Western and Eastern elements. Artists like Shankar also contributed to reimagining classical traditions, drawing on Western experimental approaches while preserving the core elements of Indian music. The global flow of music in the modern era has resulted in further convergence, with musical styles borrowing elements from each other, leading to a blending of rhythmic patterns, melodic structures, and instrumentation.

However, colonization and globalization also contributed to the preservation of distinct musical traditions. Despite the influence of Western music, both Indian classical and Chinese classical music have retained their unique characteristics, such as the use of complex rhythmic cycles (tala in Carnatic music) and modal scales (raga in Hindustani music), which set them apart from Western tonal systems. While Westernization introduced orchestral arrangements and harmonic complexity, Eastern music has resisted complete assimilation, retaining its emphasis on improvisation and non-harmonic structures.

These forms of musical resistance to Westernization helped maintain the integrity of Eastern music, even as elements of Western styles were integrated. This resistance also demonstrates the resilience of cultural traditions, as Eastern musicians and composers intentionally sought to preserve the authenticity of their art forms while adapting to external influences.

To summarize, historical events like colonization and globalization have played dual roles in the musical evolution of Eastern and Western traditions. They facilitated the blending and merging of musical elements, leading to moments of convergence, particularly through hybridization. At the same time, these processes also fueled the preservation and reaffirmation of distinct musical traditions, ensuring that Eastern and Western music, while influenced by one another, maintained their unique identities. This dynamic interaction between convergence and resistance to change underscores the complexity of cultural exchange, as well as the ongoing dialogue between musical traditions across time and space.

Perspectives

Convergence, divergence, or independence?

In analyzing the convergence, divergence, or distinctness of musical styles between Eastern and Western traditions, several potential findings could arise based on the application of cosine similarity. If high cosine similarity scores are observed, this would suggest that cultural exchange, particularly during periods of colonization and globalization, led to a

blending of musical elements. Such a finding would be consistent with Nettl's (5) assertion that cultural diffusion and contact play an instrumental role in shaping musical traditions, especially as cultures engage in cross-cultural exchanges. For example, the incorporation of Western instruments like the violin into Indian classical music or the fusion of Western harmonic structures with Indian modal scales could result in compositions exhibiting high similarity. Such findings would indicate a significant convergence of musical styles, where both traditions began to share common features due to these historical interactions. Moderate similarity scores, on the other hand, would imply that while Eastern and Western traditions influenced each other, the core musical features—such as the modal systems of Eastern music and the tonality of Western music—remained largely distinct. As Shao and Zhou (6) point out, similarities in harmonic structures or scales may occur due to both universal perceptual mechanisms and specific cultural practices. This could suggest that, during the post-colonial period, composers may have integrated aspects of both traditions, yet still preserved their cultural musical identities. Drawing from Hofstede and Hofstede's (7) concept of "cultural software," composers may have sought to retain cultural integrity even while incorporating foreign musical ideas, leading to hybrid compositions that still reflect deep-rooted Eastern and Western values.

In contrast, low similarity scores would highlight that, despite historical exchanges, Eastern and Western music maintained their distinctiveness, especially when comparing

non-hybridized compositions. For instance, Carnatic music and Baroque music might show very low similarity, reflecting enduring differences in structure, instrumentation, and rhythm, driven by their separate cultural contexts. This divergence is particularly noticeable in the contrasting role of improvisation in Indian music compared to the structured, composed nature of Western classical music (8). The implications of these findings are significant. High or moderate similarity scores would confirm the influence of cultural diffusion, colonization, and/or globalization in fostering musical hybridization, showing that music evolves through interaction and exposure to new ideas. These findings would underscore how external influences, such as the introduction of Western instruments or harmonic ideas, reshape traditional music. In contrast, such results would echo Wong's (9) notion of "cultural entanglement," where cross-cultural exchanges lead to the emergence of new musical forms while preserving cultural traditions. Conversely, low similarity scores would suggest the resilience of Eastern music in preserving its core characteristics despite Westernization, reflecting music's role in maintaining cultural identity, particularly under cultural diffusion and/or colonial pressures.

Moderate similarity in post-colonial music would indicate that, while external influences shaped the music, there was a conscious effort to maintain cultural integrity. This tension between cultural change and preservation is reflected in the evolution of Indian classical music during British colonization (8). For instance, the incorporation of Western

harmonic structures into Carnatic music was balanced by a strong adherence to the foundational principles of raga and tala, helping retain the tradition's distinctive nature. Finally, high similarity in contemporary periods would support the notion that globalization has facilitated cross-cultural collaboration, leading to the blending of musical traditions into new, hybrid forms. This trend is exemplified in the increasing popularity of fusion genres, where Western jazz harmonies intermingle with Indian classical improvisation, as seen in the works of artists like Ravi Shankar and John McLaughlin. In summary, analyzing the similarities and differences between Eastern and Western music through the lens of cosine similarity offers valuable insights into the ways cultural exchange, historical events, and globalization have shaped the evolution of musical traditions, highlighting both the blending of styles and the preservation of distinct musical identities. As Sachs (10) has suggested in his historical analysis of music's development across East and West, the exchange of musical ideas through trade routes, colonization, and globalization has contributed to the complex landscape of musical convergence and divergence.

The question of whether cultural diffusion and contact are necessary for convergence in musical traditions or if mathematical similarities can emerge independently due to shared principles is a complex one that requires considering both the influence of external exchanges and the universal aspects of human creativity. On one hand, cultural diffusion—the exchange of ideas, practices, and artistic forms

between cultures—has undeniably played a crucial role in shaping the convergence of musical styles. Historical events like colonization and globalization facilitated the movement of musical ideas across borders, leading to the blending of Eastern and Western musical traditions. For instance, during the British colonial period, the introduction of Western instruments into Indian classical music, such as the violin, created a hybrid musical tradition, combining Western harmonic structures with Indian ragas and talas. Such examples of cultural contact suggest that convergence in musical styles often requires external influences, as these traditions did not naturally merge without interaction.

However, there is also a compelling argument that mathematical similarities in music, such as harmonic structures and rhythm, could independently emerge due to shared principles inherent in human creativity. Both Eastern and Western music, for example, use certain universal principles like consonance and dissonance, pitch intervals, and rhythmic patterns to create musical coherence. Shao and Zhou (6) argue that these similarities are not only the result of cultural interaction but also reflect underlying cognitive patterns in music perception, which may cause musical systems from different cultures to converge in some respects, even without direct contact. These principles are based on natural phenomena—such as the harmonic overtone series—which may lead to similar structures emerging independently within different cultural contexts. For example, both Western classical music and Indian classical music employ systems of tension and release, often using

scales (Western major/minor, Indian raga) that are built on similar intervallic relationships. Similarly, the use of rhythmic cycles (like tala in Indian music and time signatures in Western music) both organize musical time, albeit with different traditions and approaches. This parallel development of similar musical structures, based on shared human cognitive and perceptual mechanisms, suggests that mathematical principles like interval ratios and rhythmic patterns could lead to convergent structures even without cultural contact. This convergence in musical concepts, as Wong (9) notes, highlights how the human experience of sound and rhythm shapes music across cultures, even when these cultures evolve independently.

In this sense, convergence may not solely depend on external influence, but could emerge as a result of universal cognitive and perceptual principles that shape human musical creativity. These principles may lead to similar harmonic structures, rhythmic patterns, and even formal compositional techniques, regardless of geographical or cultural boundaries. For example, the development of counterpoint in Western music and improvisational techniques in Indian classical music, while culturally distinct, both stem from the human need to create complexity within a structured framework.

Therefore, while cultural diffusion and contact have undeniably facilitated the convergence of musical traditions, particularly in terms of hybridized forms, mathematical similarities rooted in shared principles of human perception and creativity could also account for

the emergence of similar musical structures across cultures. The balance between external influence and intrinsic cognitive processes calls for a nuanced understanding of musical convergence, recognizing that both cultural exchange and universal human creativity shape the way music evolves across different traditions. The relationship between cultural contact and independent musical convergence is therefore complex and likely shaped by both external influences and inherent, universal principles of music-making.

This paper demonstrates that while cultural diffusion and contact have undeniably played a significant role in the convergence of Eastern and Western musical traditions, the study also highlights how mathematical principles, rooted in universal aspects of human cognition and perception, can independently drive the emergence of convergent musical structures across cultures.. The analysis suggests that external influences, such as colonization and globalization, facilitated the blending of musical elements, leading to hybridized forms and shared techniques, such as the incorporation of Western instruments into Indian classical music. By tracing these historical influences, the study underscores the role of external factors in shaping musical evolution.

However, the paper equally emphasizes the role of inherent human cognitive principles in the development of musical similarities. These principles, including common harmonic structures, rhythmic cycles, and compositional techniques, may emerge independently across different cultures, indicating that shared

creative processes are central to musical convergence. This highlights how humans are naturally inclined to organize sound within coherent frameworks, regardless of cultural context.. Through the examination of both historical interactions and intrinsic principles of music-making, this paper concludes that the convergence of musical traditions is shaped by a dynamic interaction between cultural exchange and shared creative processes that transcend geographical boundaries. This dual perspective, combining historical and cognitive elements, offers a comprehensive framework for understanding the evolution of musical traditions.

The interdisciplinary approach that combines cultural studies and mathematical analysis offers a unique and enriching perspective on the study of music, enabling deeper insights into both the human aspects of musical traditions and the underlying mathematical structures that shape them. By integrating cultural studies with cosine similarity and other mathematical tools (appendix 1), this approach bridges the gap between the qualitative, subjective experience of music and the quantitative, objective analysis of its elements.

From the perspective of cultural studies, this approach allows us to explore how historical events like colonization, globalization, and cultural exchange have influenced the evolution of musical traditions. For instance, the British colonial period in India serves as a key example of how external forces led to the blending of Eastern and Western musical elements. It sheds light on the social, political, and cultural contexts that have led to the

blending of Eastern and Western music, revealing how music is not just an artistic expression but also a product of historical and cultural forces. The study of musical hybridization—such as the incorporation of Western instruments into Indian classical music or the fusion of Western harmonic ideas with Eastern melodic structures—highlights the significance of cultural diffusion in shaping artistic identity and facilitating cross-cultural understanding.

On the other hand, mathematical analysis introduces an objective framework for analyzing music, enabling us to quantify and compare musical structures across different traditions. By using techniques like cosine similarity and Fourier transform (appendix 1), this study allows for the measurement of similarity between musical compositions based on features like pitch, rhythm, harmony, and structure. This provides a clear, reproducible method for comparing diverse musical traditions and identifying shared principles, such as harmonic relationships, intervallic patterns, or rhythmic cycles, that may emerge independently due to universal aspects of human creativity.

The integration of subjective exploration of cultural influences and the objective analysis of musical features enriches our understanding of music as both a human and mathematical phenomenon. It encourages a more holistic view, acknowledging that musical convergence and divergence are not just a result of cultural exchange but are also shaped by shared cognitive, biological and perceptual processes. By employing both cultural and mathematical

lenses, we gain a more comprehensive understanding of how music evolves, how different traditions interact, and how seemingly distinct musical systems may have more in common than they appear.

Conclusion

Combining cultural studies with mathematical analysis not only enhances our understanding of music's cultural significance but also provides a more robust method for examining the structural similarities and differences between musical traditions. This interdisciplinary approach fosters a deeper appreciation for both the subjective and objective dimensions of music, allowing us to explore the complex interactions between culture, history, and mathematics.

Future research at the intersection of cultural studies and mathematical analysis of music offers promising avenues for exploration. Expanding the dataset to include a broader range of musical systems, such as African, Middle Eastern, or Southeast Asian music, could offer a more comprehensive understanding of the global patterns of convergence and divergence in musical structures. Additionally, incorporating more composers and compositions from both Eastern and Western traditions would allow for a more nuanced exploration of how cultural exchange and universal principles have shaped music across time and geography. Another area for future research is the exploration of additional mathematical methods to complement or refine the current approach. While cosine similarity has proven effective for analyzing structural similarities, incorporating techniques like

Fourier analysis and graph theory could provide deeper insights into musical features, such as frequency components and the relationships between melody, harmony, and rhythm. Additionally, the use of machine learning algorithms and neural networks could help uncover complex, non-linear patterns in music that might not be easily detectable using traditional mathematical methods.

Another area of potential research is the integration of computational tools that model the creative processes in music-making, allowing researchers to simulate and predict how musical styles evolve over time. These models could account for both cultural influences and inherent cognitive biases in music creation, providing a more dynamic and predictive framework for understanding how musical traditions interact and evolve.

Finally, cross-disciplinary collaborations between ethnomusicologists, mathematicians, and computer scientists could lead to the development of more sophisticated analytical tools that incorporate both the historical and the structural dimensions of music. This collaboration could yield more accurate models for understanding how cultural, social, and cognitive factors interact to influence musical forms and practices.

In summary, this paper sets the stage for future research that could expand on the current study by broadening the scope of the dataset, exploring additional mathematical methods, and developing more sophisticated computational models to capture the complexities of musical evolution. These

efforts could deepen our understanding of the diffusion, musical structure, and universal relationships between cultural creative principles, further enriching the interdisciplinary field of music analysis.

Supplementary data

Open Office™ Calc spreadsheet

References

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Appendix

Temporally different compositions were compared to investigate if there was a link between similar compositions and certain heuristic factors of selected segments of the Fourier transforms of those compositions. These compositions included The first Delphic Hymn to Apollo, 138 BCE (1), Hurrian Hymn no. 6, 1400 BCE (2), Solitary Orchid, 500-700 AD (3), Balinese gamelan, ~ 1100 AD (4), Didgeridoo (n.d) (5), Seikilos epitaph, (1-100 AD) (6), Ashir Shirim, ~ 500 BCE (7), Robertsbridge Codex, 1360 AD (8), Radif, 1900 AD (9). These compositions were selected because they represented a wide range of time period (from 1400 BCE to 1900 CE), they were widely separated across the Old World geographic region, their compositions were written down in the form of melody (rather than as prose) and they were available and easily accessible on YouTube.

First, the videos were downloaded as .mp3 files using 4K video downloader™, version 25.1.0.0196, InterPromo GMBH. Each .mp3 file was imported into Audacity®, version 3.7.1. Its Fourier spectrum was plotted using the 'Analyze', 'Plot Spectrum' buttons. All the parameters on the spectrum window were left to default. Using the 'export' button on the 'Frequency Analysis' window, the spectrum was exported and saved as a .txt file. The .txt file contained the frequency (Hz) with its corresponding amplitude (dB) in two columns. The entire .txt file was copied and pasted into Libre Office™ Calc, version 7.3.7.2, Libre Office Contributors. In this manner, the spreadsheet contained the frequency in the first column and the amplitude associated with that frequency in the other 9 columns; each column corresponding to a composition (see supplementary spreadsheet file). The graphs for the 9 compositions are shown in Figure 1.

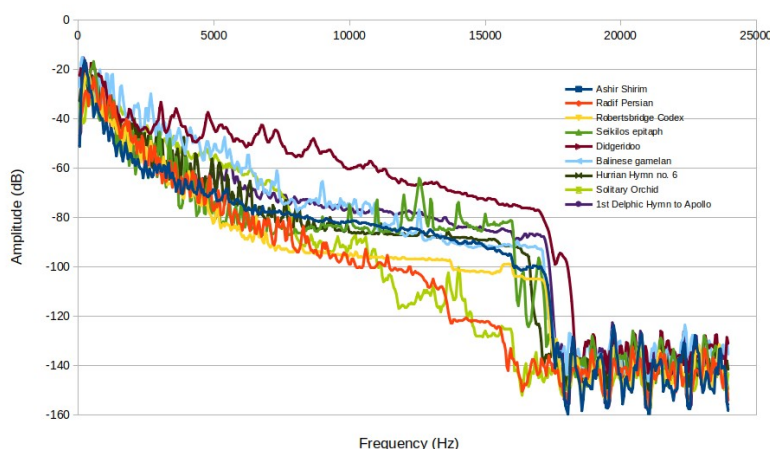


Figure 1. Fourier Analysis curves for all the compositions

Cosine similarity

To obtain cosine similarities for any two compositions, the amplitude column of each composition was appended with a comma using the CONCATENATE function (see supplementary Open Office Calc file). The two columns were then imported as two vectors into the website <https://tilores.io/cosine-similarity-online-tool> and the two compared by clicking on the compare button. The software returned a percent cosine similarity value. As shown in Table 1, the cosine similarities of compositions that diffused by different paths (see later) still returned a cosine similarity of 100%. For example, the Hurrian hymn. no. 6 and the Ashir Shirim, which are on different

paths of diffusion (Figure 5) still showed a cosine similarity of 100%. This may be because mathematization of the two FA curves as the dot product of the two vectors divided by the product of their magnitudes is not able to discern differences between different sections of the FA curves of the two compositions. It may be that a STFT is needed in conjunction with feature analysis to discern these differences. A method that is conceptually – but not mathematically – aligned with the STFT was implemented as a new approach to investigate if geographic routes for music diffusion could be be charted or predicted based on the entire Fourier Transform of particular compositions.

Table 1. Using cosine similarity to compare different compositions FA curves, the colored font can be interpreted after a read of the entire appendix

Composition 1	Composition 2	%cosine similarity
Ashir Shirim (Iraq, Babylon)	Hurrian hymn. no. 6 (North Syria)	100
Balinese gamelan (Bali)	Hurrian hymn. no. 6 (North Syria)	100
Solitary Orchid (Beijing, China)	Hurrian hymn. no. 6 (North Syria)	100
First Delphic Hymn. To Apollo (Athens, Greece)	Hurrian hymn. no. 6 (North Syria)	100
Robertsbridge Codex (Italy)	Hurrian hymn. no. 6 (North Syria)	100
Radif, Persian (Iran)	Hurrian hymn. no. 6 (North Syria)	100

Comparing selected segments of the FA curves heuristically

Another method to find similarities and differences between the various compositions was constructed. It was noticed that each Frequency-Amplitude (FA) curve corresponded to ~ four regions (as shown in Figure 2). The shape and curve of region BC was reproducible among all the compositions, hence it was processed further by having the Libre Office™ Calc software calculate the best fitting power curve ($y=ax^b$, where a =coefficient and

b =exponent) for this segment for all compositions. The 9 power equations fit their corresponding curves with moderate accuracy, having regression coefficients (R^2) ranging from 0.65 to 0.95 (average = 0.83). The exponents of these power equations were hence defined as the first heuristic feature of individual compositions. The exponent of the equations (rather than their coefficients) were chosen because the exponent influences the shape of the function as well as its responsiveness to changes in inputs. The

coefficient – by contrast – only scales the function, defining its amplitude. Subsequently the ratio of the [difference between the absolute value of point B (greatest dB) and the absolute value of point E (least dB)] to the absolute value of point B (termed as the 'normalized span', was calculated for each of the compositions. This was taken to be the second heuristic feature of individual compositions. As an example, the expression $\frac{|158.16| - |51.22|}{\text{abs}\{51.22\}} = 2.09$ was calculated for the Ashir Shirim composition (see attached spreadsheet). Table 2 shows the coefficient a, exponent b and the normalized span values for the various compositions along with their ~ historial dates of formulation.

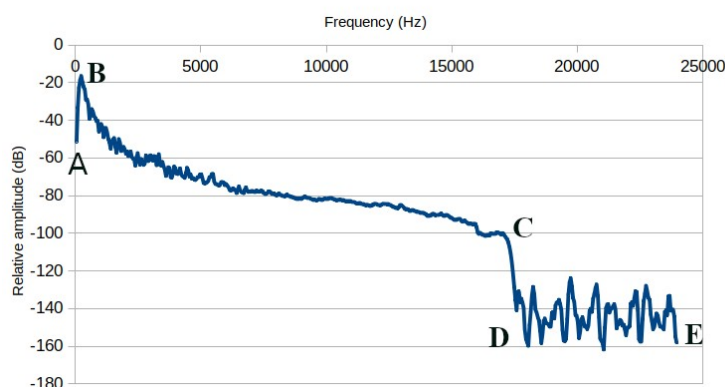


Figure 2. Regions of the Frequency-Amplitude graphs for a representative composition (Ashir Shirim, Babylon, ~ 500 BCE)

Table 2. The heuristic factors deemed to be important for defining the musical composition according to the historical date of its formulation (see supplementary spreadsheet for calculations)

Composition name and approximate location	Approximate date	Coefficient (a)	Exponent (b)	Normalized span
Ashir Shirim (Iraq, Babylon)	-500	-2.61	0.398	2.09
Radif (Persian) (Iran)	+1900	-1.94	0.423	2.19
Robertsbridge Codex (Italy)	+1360	-2.2	0.412	2.43
Seikilos epitaph (Athens, Greece)	+100	-1.34	0.472	2.63
Didgeridoo (Australia)	n.d	-3.5	0.311	4.83
Balinese gamelan (Bali)	+1100	-3.47	0.312	3.97
Hurrian Hymn. no. 6 (Northern Syria)	-1400	-2.24	0.394	3.31
Solitary Orchid (Beijing, China)	+600	-6.39	0.25	3.27
First Delphic Hymn. To Apollo (Athens, Greece)	+138	-1.89	0.432	2.33

For graphing purposes, all BCE dates are negative, all CE dates are positive.

When the historical dates on the X-axis were plotted against the exponent b , or the normalized span, the resultant graphs, shown in Figures 3 and 4, did not seem to be remarkable. The points were scattered with no readily discernible correlation between the dates of the compositions and either their FA attributes or their normalized spans. In Figure 3, the points do seem to be segregated into two groups, i.e. the four data points at the top left and the four at the bottom right. In figure 4, even such a clustering is not immediately noticeable, although a K-means or a similar clustering algorithm could conceivably find clusters. If the assumption of a monolithic diffusion of musical knowledge (a hierarchical model originating from a single source) is abandoned, instead finding mutually exclusive multiple origination modes of diffusion acceptable; it becomes evident that both

Figures 3 and 4 may be each interpreted as containing two 'routes' of musical diffusion as shown in Figures 5 and 6. In figure 5, the two routes of diffusion that were ~ orthogonal to one another were chosen (rather than two approximately parallel lines, one through the left top cluster and one through the bottom right cluster). This was done because of two reasons, 1] The heuristic factor of normalized span in Figure 6 supported ~ orthogonal lines of best fit, similar to the orthogonal model in Figure 3, and 2] orthogonality in dimensionality reduction algorithms such as Principal Component Analysis is associated with maximizing the variance of the data, therefore the variation in the heuristic attributes of the Fourier Transforms of the compositions was thought to be maximized by orthogonal arrangement of the data points.

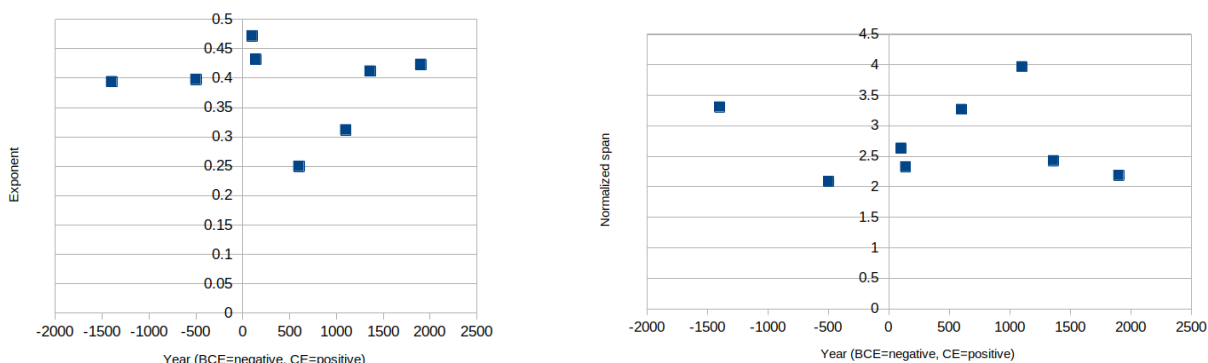


Figure 3 (left pane) and **Figure 4** (right pane). Is music independently constructed by different cultures or has it diffused in a historically hierarchical manner, with the oldest culture diffusing this knowledge to subsequent newer ones ?

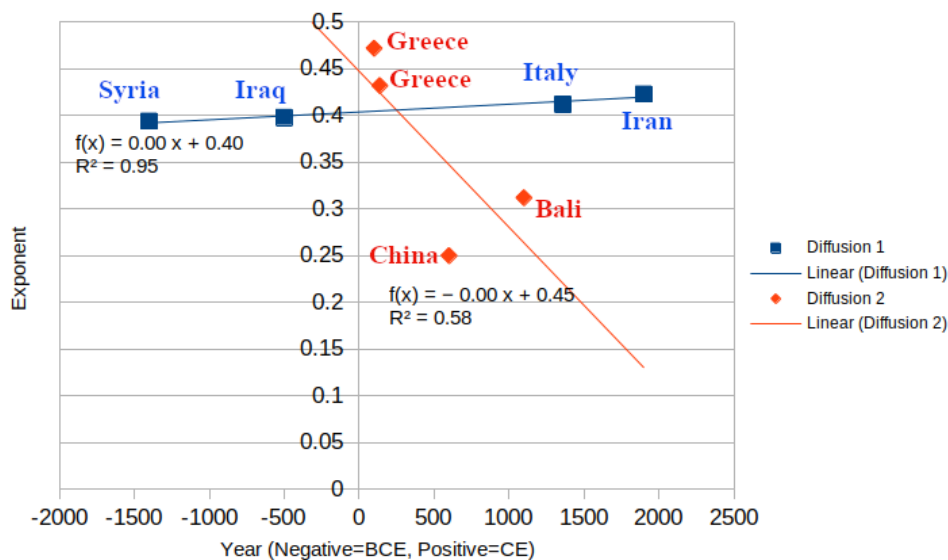


Figure 5. Two routes of musical diffusion using the exponent similarity patterns in a selected segment of the Fourier transform of the composition. One originates in Hurrian culture in Northern Syria and spreads via Anatolia and the Levant to the East to Mesopotamia and Iran and to the West to Rome. The other originates in Greece and diffuses in a north-eastern sweeping arc to China and thence on to Bali.

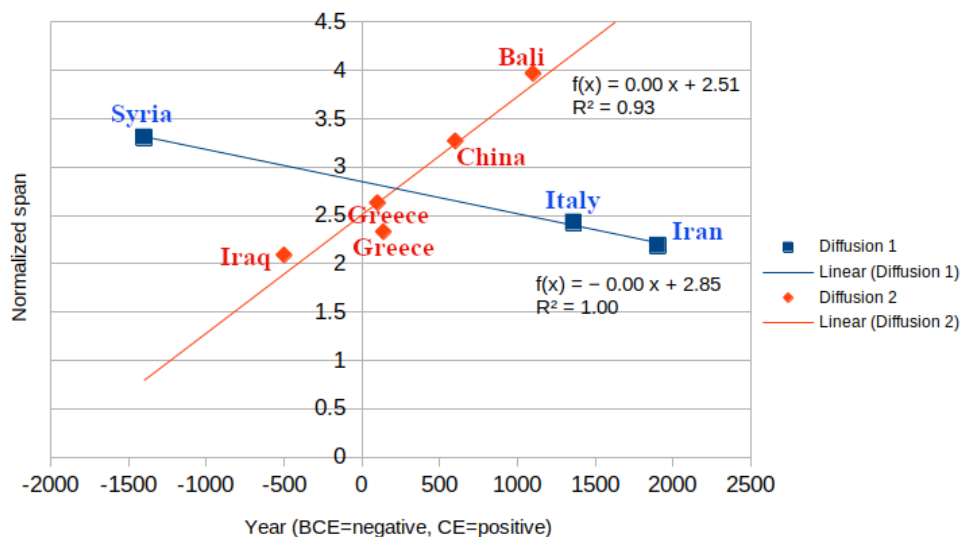


Figure 6. Two routes of musical diffusion using the normalized span similarity patterns in the Fourier transform of the composition. One originates in the Hurrian culture in Northern Syria and spreads via Anatolia and the Levant in the East to Iran and in the West to Rome. The other originates in Babylon (Iraq) and diffuses to the west to Greece and to the East to China and thence on to Bali.

An interesting challenge is presented by the composition played on the didgeridoo wind instrument, for which no origination date was found. If a requirement is imposed on the composition to satisfy both its exponent (0.311) and the normalized span (4.83) attributes, then the only way would be to go back in time (since going forward in time in Figure 6 would result in a value in the future for a normalized span of 4.83; an obvious absurdity). Substituting in the blue diffusion 1 path equations in both figures 5 and 6 such that $R^2 > 0.93$, hence yields a calculated origination date for this composition of 5000 BCE and 3000 BCE respectively. If, on the other hand, only the exponent term needs to be satisfied as per Figure 5, that leads to an origination date along the red diffusion 2 line of ~ 1000 CE for the didgeridoo instrument. In this connection, an important point must be emphasized; use of mere functionality of an instrument (including the larynx) to produce sounds (such as those produced for calling, warning, obtaining attention; among others), without the explicit intention of those sounds being epicurean, does not qualify as the beginning of music. Therefore, even though the didgeridoo may have been used longer than when it was first inscribed into rock-art (~ 20 CE), there is no consensus and no written record of when it was actually harnessed for making melodious sounds. Note that there is an ethnocentric bias in making the aforementioned argument, since a 'melody for one may be cacophony for another'.

Discussion

If similarity in music, primarily measured using its Fourier transform, is correlated with the

heuristic attributes of the exponent of the fitted function to a selected segment of the FA curve and/or to the normalized span; then it follows that musical compositions originating and/or diffusing from/through Greece, China and Bali are similar and belong in one group, while those originating and/or diffusing from/through Syria, Iraq, Italy and Iran are similar and belong to another group (as interpreted using Figure 5).

Figure 6 similarly (but not congruently) shows that musical compositions originating and/or diffusing from/through Iraq, Greece, China and Bali are similar and belong in one group, while those originating and/or diffusing from/through Syria, Italy and Iran are similar and belong to another group.

Regardless of which clusters are chosen, it is interesting to note that – if the time-spread diffusion of music thesis is adopted – there appear to be two originators and/or nodes of diffusive spread, which – in itself – is interesting. This is because it seems incongruous that overlapping geographical regions may somehow be 'left out' in this temporal diffusion process. For example, from Figure 6, why would Greece be bypassed by music diffusing from Syria to Italy? Similarly, why would Iran be bypassed by music diffusing across the Iraq – Greece – China- Bali geographical trajectory? Even if mathematical similarities in music were hypothesized to independently emerge (rather than diffuse) among different cultures, why would cultures or regions that were supposedly interconnected via trade routes and empire building activity – even though temporally apart – develop

different mathematical models of music ostensibly uninfluenced by civilizational contact ? Assuming that Greece was an intersection of diffusion paths 1 and 2, another observation could be made that music ideation spread from Syria to Greece, then geographically *backtracked* to Iraq, China and Bali (although this would not align with the main hypothesis of patterns of the Fourier transform attributes correlating with time). This observation does not make geographical sense

either, unless, like Zheng He's expeditions from 1471-1472 (as interpreted prior to revisionist history); the migrants perceived slim chances of civilized peoples existing west of Greece. Of course, it is obvious that not all the population of a culture moves and/or migrates at any given time, hence migration routes may originate from the same culture toward different directions, regardless of whether or not these migrations were temporally separated.

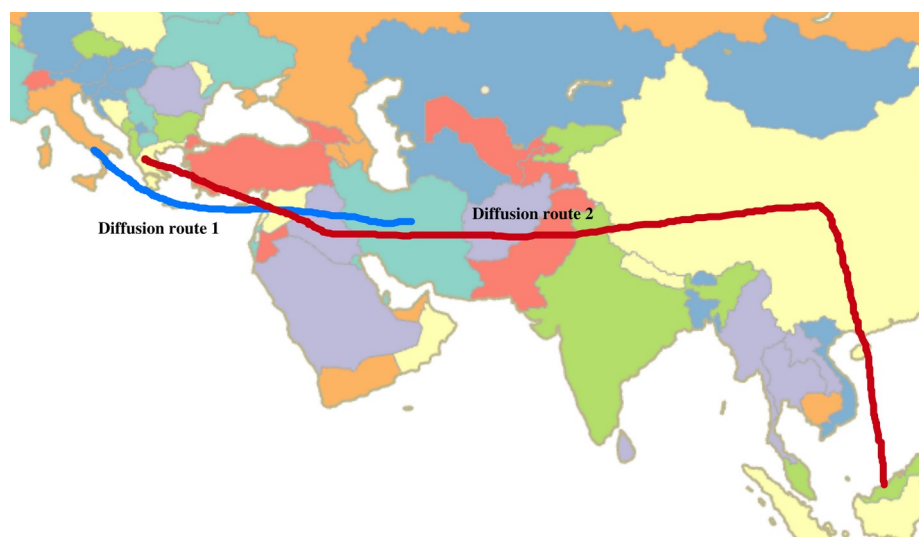


Figure 7. A geographical representation of music diffusion routes according to data in Figure 6. The Origination of Diffusion route 1 is Northern Syria and that of Diffusion route 2 is Babylon (Iraq). The musical compositions comprising Diffusion route 1 display a different time dependent feature pattern than the musical compositions comprising Diffusion route 2.

Limitations

The challenge in this method is that there may be insufficient or undiscovered written records of music for the various civilizations studied. It may also be that musical knowledge was passed down from generation-to-generation without the use of the written medium, may have been destroyed over time, or may not have been deciphered as yet. The method also

assumes that because certain heuristic attributes of certain compositions are similar, the oldest civilization in the group is the originator of that particular musical style; i.e.; there is an implicit assumption of historical ideative hierarchy and subsequent diffusion of music.

The origins and dates of formulation of these compositions are approximations deciphered

from available evidence. Even though the written organic material can be dated using Radiocarbon-14 dating or similar, there is no assurance that the date on which the composition was written was actually the date on which it was conceived or ideated. There are differences in opinion and not a definite scholarly consensus on the origin of certain compositions. For example, the Robertsbridge Codex has now been revised to have originated in England even though three of its six pieces are written in the form of the estampie, an Italian dance form of the Trecento, and two out of the remaining three arrangements of motets are from the Roman de Fauvel. There are also invariably bound to be differences between how the original composition was conceived and how its translation is presented in contemporary media; including on YouTube – the 'Lost in Translation' effect.

Simply because the historical dates on which compositions were transcribed correlate with the exponent of the power function fitted to a heuristically chosen segment of the Fourier transform, or the normalized span of those compositions, does not imply that there may not be other (more relevant and/or non-linearly related) factors/attributes of the Fourier transforms that may more accurately represent origination and/or diffusion. These factors may be better grouped into 'clusters' using Machine Language algorithms, wherein, the conclusions drawn from this analysis would be incorrect.

The discussion and conclusions drawn from this method are based on an analysis of eight compositions (out of an expected innumerable compositions) that have probably evolved in

these civilizations. This extremely small sample cannot be considered to be representative of their respective cultures and the observed correlations may have occurred purely by chance.

Conclusion

Certain heuristically selected components of Fourier transform curves of the musical compositions from geographically spread cultures were found to be correlated with the date of the origination of those musical compositions, suggesting that music may have originated in two cultures independently and spread over time to other cultures. Although this conjecture is difficult to defend in the presence of substantial caveats (see Limitations), the strength of the correlation for both orthogonal origination and diffusive paths (see Figure 6) was large ($R^2 > 0.93$); a phenomenon which is uncommon in as factor-rich and as factor-confounding a discipline as ethno-musicology. The analysis suggests that geographical spread of the features of music composition from an originator culture(s) to other cultures cannot be definitively discounted. It is however not possible to state with any degree of certainty whether the originator culture imposed its music on a music-devoid recipient culture or whether an already independent music from the recipient culture assimilated enough features of the originator culture so as to produce a time dependent pattern similarity in their cultural musical signatures. Regardless; (unless coincidental), the temporal correlation between certain heuristic features of the Fourier Transforms of culture-specific musical compositions is suggestive of cultural

exchange, diffusion or contact between civilizations as a pre-requisite to the formation and evolution of music

Appendix References

1. <https://www.youtube.com/watch?v=eXYa6FLxRrY>
2. <https://www.youtube.com/watch?v=1Tn9x-IDYcY>
3. <https://www.youtube.com/watch?v=0ZhMeRUNpIU>
4. <https://www.youtube.com/watch?v=-0AkXdP-LMA>
5. <https://www.youtube.com/watch?v=yG9ZX1FS20A>
6. <https://www.youtube.com/watch?v=U8Y7hon911Q>
7. <https://www.youtube.com/watch?v=Tow8uBj0C8Y&t=38s>
8. <https://www.youtube.com/watch?v=c05RQnncS6s>
9. <https://www.youtube.com/watch?v=BQxfO7q5sSg>