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The influence of different factors on the maximum height reached of a water powered rocket.

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Abstract

The influence of the initial air pressure and the volume of the water on the maximum height of a 2 liter soda bottle rocket were determined. Pressure exerted a larger effect on the maximum height than the water volume. This could be attributed to a greater pressure head and a decreased time of water depletion, t_{wd}, analogous to increased water acceleration; with increasing pressures. The maximum experimental height reached by the rocket was found to be a function of the final pressure in the rocket calculated from Boyle's law and the t_{wd} calculated from first principles.

Introduction

The time of flight of water rockets - and consequently, the distance of travel against gravity - can be well described by equations of motion, fluid dynamics and thermodynamics. However, this theoretical time rarely equates to what is observed experimentally, due to assumptions that assume ideality in the theoretical calculations. Consequently, only by performing experiments can the maximum height reached by a water rocket of specific dimensions operated at specified factors (such as pressure and water volume) be determined. If there were to exist a correlation between theoretical and experimental values, *via* a function that would incorporate only the dimensions and operating factors; it would enable *a priori* prediction of water bottle rocket height for bottles of any dimension at any operating factors by performing only one set of experimental measurements. Such a description requires adoption of the design of experiments (DoE) protocol by which the relative influence of independent variables - either by themselves, or in combination - on the dependent (response) variable - the maximum height reached - can be described within predefined experimental parameters.

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Materials and Methods

A 2 liter soda bottle was used for the rocket body. 3 fins were made from plastic (cut from a plastic file folder) in the shape of right angled triangles with dimensions 3.25 inches, 1.5 inches and 3.6 inches for the two sides and hypotenuse respectively and attached to the rocket using
hot glue. The fins help to increase the rocket's stability, and maximize the height by allowing the rocket to keep a relatively straight trajectory. A nose cone was prepared from another 2 liter
bottle, by cutting off the bottom section and keeping the top, it was then attached to the rocket. The dimensions of the rocket are presented in Table 1. Two feet long sections of 1/2 inch PVC tubing were joined together using PVC primer and cement to create a launch pad for the rocket.

The PVC pipe that was to be inserted into the soda bottle was heated and manually compressed by pushing the two ends together such that it formed a lip immediately beneath the soda bottle opening (figure 5) thereby preventing escape of the water from the inverted bottle
until the desired pressure was reached and the rope pulled. Eight inch long zip ties covered with a 1 inch piece of PVC pipe were used to hold the rocket in place until a rope attached to the 1 inch PVC pipe tubing was pulled. The end of the 30 inch long PVC pipe projected into the air space inside the rocket to enable pressurization. Upon pulling down the 1 inch PVC pipe piece with the rope, the zip ties disengaged from the bottle nozzle lip allowing for take off. The time of flight of the rocket was measured using a smartphone stopwatch. A bicycle pump equipped with a pressure gauge was attached to one end of one of the PVC pipes thereby allowing pressurization of the entire apparatus; and of the air above the water in the bottle to a predetermined value. Figures 1 through 5 show details of the launch pad assembly and mechanism of pressurization and take off. References 2 and 3 link to assembly videos.

Figure 1

Figure 2

Figure 3

Figure 4 Figure 5

The graphs used to predict the time of water depletion, t_{wd}, were obtained from the website in reference 1.

Table 1: DImensions of the 2 L soda bottle water rocket

Diameter of the rocket body (soda bottle), d	0.1110 m
Length of the rocket body (without nose cone), L	0.3300 m
Length of the nose cone	.0254 m
Cross sectional area of the rocket nozzle, A	3.878×10^{-4} m^2
Maximum water volume inside rocket body, V_w	2.000×10^{-3} m^3
Cross sectional area of rocket body, A_c	0.011 m^2

The other variables used in the equations that describe the rocket motion are as follows:
T= thrust exerted on the rocket created by the exiting water, u = velocity of the exiting water relative to the rocket, m = mass of the rocket plus the water, t = time, k = Poisson Constant, (1.4), P_0 = initial absolute pressure of air inside the rocket body before launch, V_a = initial volume of air inside the rocket body before launch, ρ_w = density of water (1000 Kg/m^3), C_d = drag coefficient (0.8), ρ_a = density of air (1.2 kg/m^3), g = acceleration due to gravity (9.8 m/s^2), atmospheric pressure (101325 N/m^2).

The Drag Coefficient value of 0.6 was calculated from the equation presented on page 393 in reference 4, however the java applet had fixed values of the drag coefficient. Therefore a value of 0.8 was chosen, it was found that there was no difference in the value of t_{wd} using either the value of 0.8 or 0.2 indicating that for the purposes of this paper a drag coefficient of 0.8 was satisfactory.

The DoE was modeled with a two factor, three level (3^2) full factorial design. The two factors were, the volume of the water inside the rocket, and the initial pressure of the air in the rocket. The three levels investigated were low, medium and high. A total of nine experiments were performed, each with a minimum of two trials as shown in table 2 below. Data analysis was performed using Minitab version 6.11, Minitab Inc., PA, USA.

Table 2: Design of experiments

Experiment No.	Volume (mL) of water in rocket	Initial pressure (psig) of air in rocket
1	Low	Low
2	Low	Medium
3	Low	High
4	Medium	Low
5	Medium	Medium
6	Medium	High
7	High	Low
8	High	Medium
9	High	High

water volumes(mL), low = 350, medium = 800 and high = 1250
Pressure(psig), low = 20, medium = 30, high = 40

Results and Discussion

It was not possible to change the relative dimensions - such as the bottle diameter, nozzle diameter, length of bottle - of the soda bottle. The factors that could be changed were the initial pressure, the fluid volume, the fluid temperature and the fluid density. Due to availability and environmental constraints, higher density fluids such as glycerin were not used. The density of water decreased by 2.8% when going from 20°C to 80°C, thereby negating this as a factor in influencing the rocket height. Therefore, the factors that were capable of manipulation and could affect the maximum height reached by the water bottle rocket were the initial pressure and the initial water volume in the bottle rocket. These factors were studied between the levels of 20 psig to 40 psig for the pressure and 350 mL to 1250 mL for the water volume.

The situation where the most *efficient* use is made of the pressurized air in propelling the rocket is one where the air inside the rocket reaches atmospheric pressure (15 psig) just as all the water volume has exited the rocket body. Note that this situation does not necessarily equate to the greatest height reached by the rocket because the thrust exerted by the air on the nearly depleted water is negligible; consequently, the thrust exerted by the exiting water on the rocket body decreases at a greater rate than if the final pressure of the air in the rocket at full water depletion were to be greater than atmospheric pressure. The pressure head forcing out a given volume of water per sec^2 could be formally considered analogous to the acceleration Psig/$(t_{wd})^2$ of the water leaving the rocket. This was useful in correlating theoretical equations for predicting maximum height to the experimentally determined height. The volume of water corresponding to a particular air pressure for the most efficient use of the pressurized air can be calculated using Boyle's law as follows:

$P_i V_i = P_f V_f$, where P_i is the (predetermined) initial air gauge pressure in the rocket, P_f is the final pressure immediately upon water depletion (15 psig), V_f is the final volume occupied by the air immediately upon water depletion (2000 mL for the 2 L soda water bottle rocket) and V_i is the initial volume that need be occupied by the air at pressure P_i. Using Boyle's law (assuming adiabatic expansion), the most efficient initial air volume - and hence the initial volume of water in the rocket can be calculated for different initial air pressures as presented in table 3.

Table 3: Initial pressure and air volume for most *efficient* utilization of pressurized air

P_i (psig)	V_i of air (mL, calculated)	P_f (psig)	V_f of air (mL)	Time for water depletion, t_{wd} (sec)	Calculated maximum height (m) reached by rocket[1]
20	1500	15	2000	0.09	17.70
30	1000	15	2000	0.099	18.82
40	750	15	2000	0.1	18.77

1. Calculated as follows: t_{wd} was obtained from the applet in reference 1 (more details appear below). The equation in figure 7 was solved for x = (P_f x t_{wd})

As is evident from Table 4, for a given volume of water in the rocket, as the pressure increased the maximum height that the rocket reached increased, while the calculated water depletion time, t_{wd} decreased. When similar pressures were compared across different water volumes in the rocket, it was found that when the water volume was 800 mL (medium), the rocket traveled to the highest maximum height. Since, the pressure head forcing out a given volume of water per sec^2 could be considered analogous to acceleration, when the pressure head (obtained by subtracting the higher of the final pressure or 15 psig from the initial pressure) was divided by the square of t_{wd}, it was indeed found that - with the exception of the case of medium volume and low pressure - the acceleration was greatest when the water volume was 800 mL. This explained why the water rocket traveled to maximum heights at this water volume.

Table 4

Experiment No.[1]	Volume (mL) of water in the rocket	Initial Pressure (psig)	Time of Flight[3] , T (sec) ± RSD%	Maximum Height reached[4] (m)	Calculated pressure[5] of air at 2 L (psig)	Calculated water depletion time[6] (sec)
1	Low	Low	3.51 ± 0.87	15.09	16.5	0.077
2	Low	Medium	3.77 ± 1.51	17.41	24.8	0.063
3	Low	High	3.88 ± 2.55	18.44	33.0	0.055
4	Medium	Low	3.73 ± 1.7	17.04	12.0	0.114
5	Medium	Medium	3.94 ± 0.00	19.02	18.0	0.091
6	Medium	High	4.24 ± 3.33	22.02	24.0	0.079
7[2]	High	Low	2.10 ± 10.4	5.40	7.5	0.153
8	High	Medium	3.64 ± 0.96	16.23	11.3	0.113
9	High	High	3.93 ± 0.36	18.92	15.0	0.100

1. For each experiment, number of trials were 2 or 3. RSD% = relative standard deviation%
2. For experiment 7, water was found in the rocket upon landing.
3. The time to reach maximum height (sec) is half the time of flight.
4. Calculated as $(timeofflight/2\ x\ g)^2/(2g)$
5. Calculated from Boyle's law using P_i=initial pressure, V_i=initial air volume over water in the rocket, V_f=2000 mL
6. Calculated from the graph of thrust versus time from the Java applet in reference 1 with a representative screenshot in figure 8.

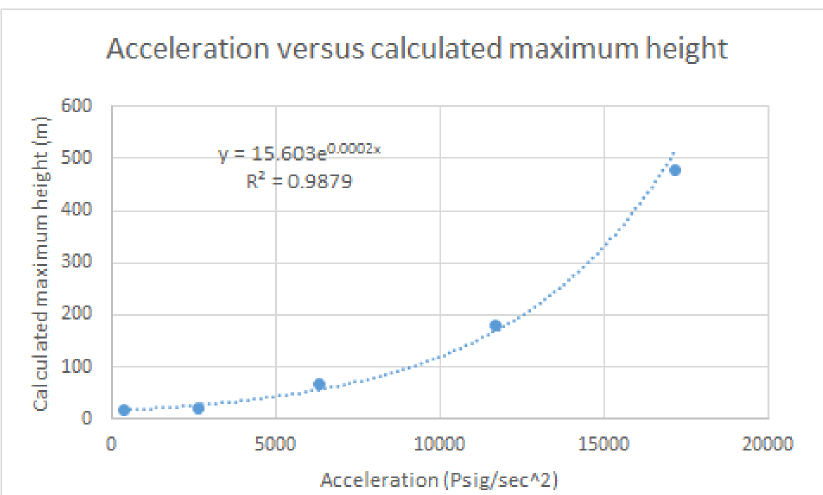
As seen from table 4, the t_{wd} was lesser by up to two orders of magnitude when compared with the time taken to reach the maximum height. As discussed above, a greater pressure head generated a smaller t_{wd} thereby increasing the acceleration (psig/sec^2) , which, in turn, increased the maximum height reached by the rocket. Table 5 shows calculated t_{wd} values for a given water volume at different initial pressures. There was an exponential relationship between the acceleration and the experimentally observed maximum height with a R^2 >0.98 as shown in figure 6.

Table 5

Initial pressure (psig)	Time for water depletion[1], t_{wd} (sec)	Calculated[2] final pressure from Boyle's law (psig)	Calculated[3] Pressure head (psig)	Acceleration[4] (psig/sec^2)	Calculated[5] maximum height (m)
20	0.11	12.4	5.0	413.22	17.99
40	0.076	24.8	15.2	2631.6	21.33
60	0.06	37.2	22.8	6333.3	67.96
80	0.051	49.6	30.4	11687.8	180.66
100	0.047	62.0	38.0	17202.4	479.20

1. Calculated from the Java applet in reference 1.
2. For an initial water volume of 760 mL.
3. Calculated by subtracting the greater of the final pressure or 15 from the initial pressure.
4. Calculated as [Pressure head/$(t_{wd})^2$]
5. Calculated from the equation in figure 9.

Figure 6



P values of .176 and .096 suggests that the maximum height of the water bottle rocket is affected by the water level and the initial pressure at confidence levels of 82.4% and 90.4% respectively. The DoE results presented in figure 7 indicate that the effect of pressure and water volume (height) significantly influence the maximum height achieved by the rocket at p levels < 0.1 and 0.18 respectively. It follows that the initial pressure has a greater effect on the maximum height reached by the rocket than does the initial water volume.

Figure 7: DoE results

Source	DF	Seq SS	Adj SS	Adj MS	F	P
height of water	2	51.804	51.804	25.902	2.77	0.176
initial pressure	2	83.500	83.500	41.750	4.47	0.096
Error	4	37.401	37.401	9.350		
Total	8	172.704				

S = 3.05781   R-Sq = 78.34%   R-Sq(adj) = 56.69%

Least Squares Means for max height reached

height of wa	Mean	SE Mean
1	16.98	1.765
2	19.36	1.765
3	13.52	1.765
initial pres		
1	12.51	1.765
2	17.55	1.765
3	19.79	1.765

Figure 9 shows the correlation between the final pressure (as calculated from Boyle's law in table 4, column 6 multiplied by t_{wd} (as calculated from the Java applet from the graph of thrust versus time) versus the experimentally observed maximum height reached by the rocket. The steps for predicting the maximum height reached by the water rocket, at any determined pressure and water volume, within the experimental parameters studied can be listed as follows:

Step 1. From the known pressure to be applied and the known quantity of water to be filled initially in the rocket, calculate the final pressure from Boyle's law as shown in figure 4, column 6.

Step 2. Input the pressure and water quantity parameters into the Java applet and determine the time for water depletion (t_{wd}), from the graph of Thrust versus time generated by the applet. Figure 8 shows a representative screenshot with the initial pressure = 20 psig and the volume of water = 350 mL. It can be seen that the t_{wd} = 0.077 sec, congruent to the point where the thrust begins to decrease from a maximum in the graph.

Step 3. Multiply the values from steps 1 and 2. This value, x, is then inserted into the equation in figure 9 to find the value, y; which is the maximum height reached by the water rocket.

It is evident that the data in figure 9 is best described by a third degree polynomial. This is because the x axis function incorporates both the pressure and water volume factors (as represented by the final pressure) which are varied across 3 levels (low, medium and high). As a result, the nine data points represent three discrete data sets for a given factor which show the same trend but with overlapping magnitude. Incorporating them into a single graph hence requires a third degree polynomial function as a best fit curve with an $R^2 > 0.97$.

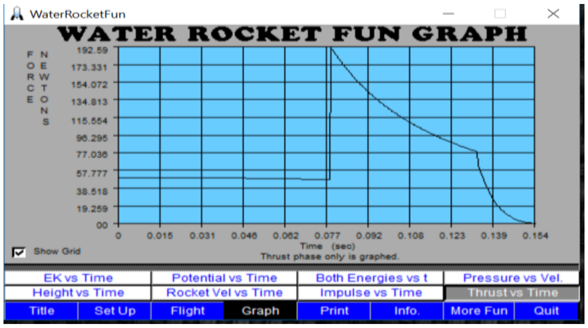
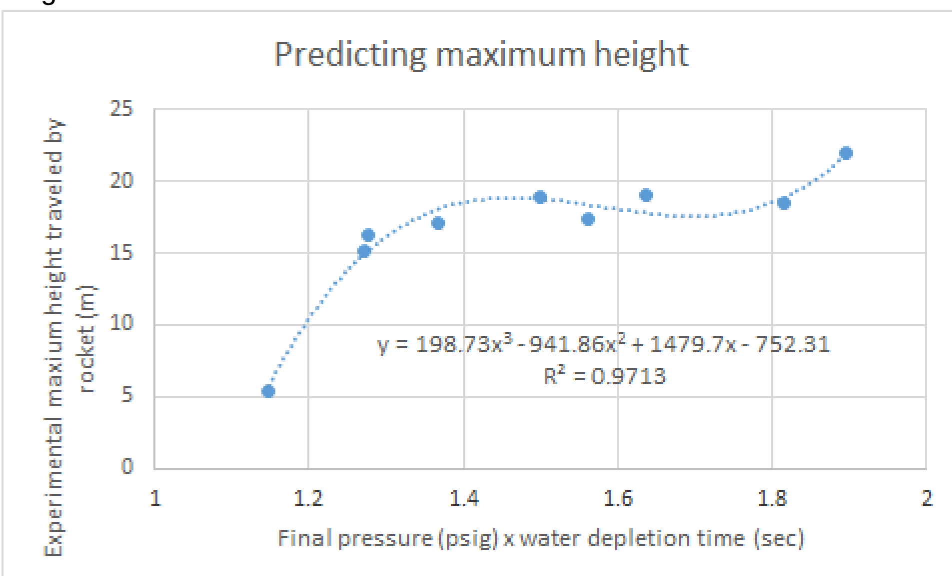


Figure 8: screenshot of Java applet

Figure 9: Predicting the maximum height reached by the bottle rocket at any pressure and water height



Conclusions

The initial pressure and the water volume significantly influenced the maximum height reached by a 2 L soda water bottle rocket within the pressure limits of 20 to 40 psig and water volume limits of 350 to 1250 mL. The DoE showed that the effect of pressure was much greater than that of the water volume in influencing the maximum height. The maximum height at any pressure and water volume within these limits could be calculated by using an empirical function that combined the initial pressure, water volume and theoretical water depletion time and plotting it against the experimentally observed maximum height reached. The ratio of pressure head to $(t_{wd})^2$; analogous to acceleration, was directly proportional to the calculated maximum height reached for the water rocket. Consequently, using first principles alone, this combination of factors can be calculated for a water rocket of any size and/or dimensions so as to obtain the greatest height from one set of experiments performed with a bottle of given dimensions.

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